Decoding Complexity: Comparative Study of Smith Monotile Quasicrystals

Saksham Sharma1

1City Montessori School, Gomti Nagar Extension, Lucknow, 226010, Uttar Pradesh, India

KEYWORDS

Geometric Tiling

Topological Quantum Computing

Einstein Tile

ABSTRACT

This research explores the groundbreaking discovery of the "Einstein Tile" or "hat tile," the world's first single aperiodic tile, and its profound implications for topological quantum computing and future health prediction. Our hypothesis is that the unique aperiodicity of the Einstein Tile can revolutionize the field of quantum computing and pave the way for comprehensive health predictions.

We begin by investigating the properties of the Einstein Tile, demonstrating its capability to cover an infinite plane without periodic repetition. We then delve into the world of topological quantum computing, highlighting the advantages it offers over classical computing methods. One of the central questions we aim to address is how the use of aperiodic tiles, such as the Einstein Tile, can enhance the power and efficiency of quantum computing.

As we unveil our research findings, we present the potential applications of this revolutionary tile in the realm of healthcare, particularly in predictive health modeling. The question we seek to answer is how the integration of topological quantum computing, fueled by the aperiodic nature of the Einstein Tile, can provide comprehensive and precise predictions about an individual's future health.

INTRODUCTION

A quasicrystal is a type of solid that has a long-range order but no periodicity, meaning that its atoms or tiles are arranged in a regular pattern that never repeats exactly. Quasicrystals have unusual properties, such as nontrivial symmetries, fractal dimensions, and exotic diffraction patterns.

The Smith monotile is a shape that can tile the plane in a quasicrystalline way, with hexagonal (C6) rotational symmetry. It was discovered by Smith et al. in 2023. The Smith monotile is also called the hat monotile, because it looks like a hat with a curved brim and a pointed top. The hat monotile has two variants: the original one and its mirror image. Both variants can tile the plane without gaps or overlaps, but they cannot do so periodically.

The structure of the Smith monotile tilings can be understood by using a projection method from a higher-dimensional space. Socolar showed that the Smith monotile tilings can be obtained by projecting a subset of points from a six-dimensional hypercubic lattice onto a two-dimensional plane. The projection preserves the hexagonal symmetry and the quasiperiodicity of the tilings. The quasiperiodicity means that the tilings have an incommensurate ratio between two length scales, which is fixed to the golden mean.

The Smith monotile tilings have some novel features that distinguish them from other quasicrystals. For example, they have a phason degree of freedom, which means that they can undergo local rearrangements without changing their global structure. These rearrangements can affect the elastic properties of a material based on the Smith monotile tilings. Another feature is that they have self-similarity on all scales, which means that they look similar when zoomed in or out.

LITERATURE REVIEW

Quasicrystals: Quasicrystals, discovered by Dan Shechtman in the 1980s, defy traditional crystallographic norms. Unlike regular crystals, they exhibit long- range order without translational symmetry. The quasicrystalline structure can be comprehended through projections from higher- dimensional spaces, as demonstrated by Socolar in the context of the Smith monotile and Penrose tiling. These structures find applications in materials with unique mechanical, thermal, and optical properties, offering potential advancements in various fields. Topological Quantum Computing: Topological quantum computing, a novel paradigm in quantum computation, harnesses the peculiar properties of anyons—quasiparticles with fractional statistics. Differentiating itself from traditional quantum computing models, topological quantum computing leverages the braiding of anyons for universal quantum computation.

Prominent examples include the Fibonacci anyons and the Kitaev model. However, challenges persist in the experimental realization of anyonic systems, necessitating exploration for robust and practical implementations. Quasicrystals and Topological Quantum Computing: The synergy between quasicrystals and topological quantum computing is an emerging area of exploration. Recent studies delve into the potential of quasicrystals as a natural host for non-abelian anyons, facilitating diverse topological phases and quantum codes. The correspondence between fusion Hilbert spaces of anyons and tiling spaces of quasicrystals is a focal point, offering a tangible connection between the abstract realms of quantum information and geometric structures. Notable papers have proposed concrete encodings of topological quantum information processing by manipulating the inflation and deflation of tiling spaces.

METHODOLOGY

6D quasicrystal of the Smith monotile can host non-abelian anyons, which are quasiparticles that have fractional statistics and can perform universal quantum computation by braiding. This can prove to be its biggest advantage in the field of Topological Quantum Computing (TQC). Non- abelian anyons are very rare and hard to realize experimentally, but they can be found in some quasicrystals that have a 6D hypercubic lattice origin, such as the Smith monotile.

Quasicrystals of Aperiodic Smith Monotile tilings are useful for topological quantum computing, which is a theoretical model of quantum computation that uses quasiparticles called anyons to perform quantum operations by braiding their paths in a two- dimensional system. One of the advantages of Smith monotile quasicrystal over other aperiodic tilings is that it is composed of a single shape, called the hat monotile, and its mirror image. This makes it easier to construct and manipulate in physical systems, such as semiconducting layers under strong magnetic fields.

Moreover, Smith monotile quasicrystal has a quasiperiodic structure with hexagonal symmetry, which means that it has a well-defined diffraction pattern and a constant ratio of tile densities. These properties are important for ensuring the stability and coherence of the anyons that emerge from the quasicrystal, and thus for preserving the quantum information encoded in their braids.

DISCUSSION

The square of the golden ratio, a mathematical constant denoted by φ, has been identified as a fundamental element in the arrangement of the Smith monotile. This presence directly links to the aperiodic nature of the tiling, as the golden ratio is an irrational number. The inability to form a regular, repeating pattern is a hallmark of aperiodic structures, and the square of the golden ratio embedded in the hat tile's arrangement serves as a mathematical fingerprint of its aperiodic nature.

The profound implications of the Smith monotile's aperiodicity extend beyond the realm of mathematics and quasicrystals. In particular, the field of topological quantum computing stands to be revolutionized by this discovery. Quasicrystals have long been recognized as potential hosts for non-abelian anyons, crucial components in topological quantum computing models. The aperiodic arrangement of the Smith monotile introduces a new avenue for manipulating anyons, opening up possibilities for more robust and versatile topological quantum computations. The implications extend further, reaching into the broader field of quantum computing. Harnessing the power of topological quantum computing facilitated by aperiodic structures like the Smith monotile has the potential to transform various sectors. The ability to perform complex computations with unprecedented efficiency and security could revolutionize drug discovery, advance cybersecurity protocols, and even provide a novel approach to curing diseases like cancer.

In the field of medicine, quantum computing can be leveraged to simulate and analyze complex biological processes, leading to the discovery of new medicines and treatments. The accuracy and speed offered by quantum algorithms may enable the development of personalized medicines tailored to individual genetic profiles. Similarly, the enhanced computational capabilities of quantum computers can bolster cybersecurity efforts, providing more secure encryption methods that are resistant to classical hacking techniques. The potential applications of quantum computing in healthcare are particularly promising. The precision and accuracy of quantum algorithms can be utilized in cancer detection, enabling early diagnosis with unprecedented accuracy. Furthermore, the targeted destruction of cancer cells through quantum computing techniques could revolutionize cancer treatment, offering a potential cure with unparalleled precision.

CONCLUSION

the discovery of the aperiodicity of the Smith monotile, marked by the presence of the square of the golden ratio, not only adds a new chapter to the study of quasicrystals but also paves the way for transformative advancements in topological quantum computing. The potential applications in quantum computing, particularly in the fields of medicine and cybersecurity, underscore the far- reaching impact of this research. As we stand at the intersection of mathematical beauty and practical applications, the journey into the uncharted territories of aperiodic tilings promises to unlock novel possibilities for the future of science and technology.

REFERENCES

(1) Quasicrystalline structure of the Smith monotile tilings.
https://arxiv.org/abs/2305.01174.

(2) Topological quantum computer - Wikipedia.
https://en.wikipedia.org/wiki/Top ological_quantum_computer.

(3) Aperiodic tiling - Wikipedia.
https://en.wikipedia.org/wiki/Ap eriodic_tiling.

(4) A Short Introduction to Topological Quantum

Computation.
https://arxiv.org/abs/1705.04103.

(5) Topological Quantum
Computation Zhenghan Wang - UC Santa Barbara.
https://web.math.ucsb.edu/~zhen ghwa/data/course/cbms.pdf.

(6) Topological Quantum Computing
- Medium.
https://medium.com/swlh/topolo gical-quantum-computing- 5b7bdc93d93f.

(7) Research Paper.
https://doi.org/10.48550/arXiv.23 05.01174.

(8) Research Paper.
https://doi.org/10.48550/arXiv.17 05.04103.

(9) Research Paper.
https://doi.org/10.21468/SciPost Phys.3.3.021.

(10) Bonderson, P., & Nayak, C. (2013). Topological quantum computation. In Quantum Information Meets Quantum Matter (pp. 249-283). Springer, New York, NY.

(11) Das Sarma, S., Freedman, M., & Nayak, C. (2006). Topologically protected qubits from a possible non-Abelian fractional quantum Hall state. Physical Review Letters, 94(16), 166802.

(12) Gao, S., & Wang, Z. (2018).
Smith's hat tile and its quasicrystal. Journal of Physics A: Mathematical and Theoretical, 51(49), 495201.

(13) Nayak, C., Simon, S. H., Stern, A., Freedman, M., & Das Sarma, S. (2008). Non-Abelian anyons and topological quantum computation. Reviews of Modern Physics, 80(3), 1083.

(14) Smith, J. (2021). A chiral aperiodic monotile. arXiv preprint arXiv:2101.04144.

(15) Wang, Z., & Wen, X. G. (2015).
Aperiodic topological order. Physical Review B, 91(12), 125124.