Developing Relationship Between Ambient Temperature and Main Tuning Slide Length of a B-Flat Trumpet

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Abstract

Trumpet players rehearse and perform indoors and outdoors. However, large temperature differences in various settings significantly affect pitch. This project investigates how temperature affects the main tuning slide length of a standard B-flat trumpet to produce accurate harmonic pitches, based on the equal tempered chromatic scale. Six experimental trials were conducted, in which temperature sensors and measuring tapes were attached to respective locations on the trumpet. The project report provides a model with a table of values describing the ambient temperatures and corresponding tuning slide lengths for the trumpet player to reference. They can make accurate adjustments to play in tune and enhance intonation, when located in different environments with varying ambient temperatures.

Introduction

The resonant sound a trumpet produces comes from standing sound waves. Standing waves are inverted after reflecting off a boundary, and match perfectly in frequency (pitch) and amplitude (loudness) with incoming waves. The constructive interference that results contributes to resonance. A trumpet is an open-closed pipe, with the mouthpiece as the closed end and the bell as the open end. When an odd integer multiple of quarter wavelengths fits into the length of the pipe, standing waves are produced [1]. The structure of a standard B-flat trumpet is shown in Figure 1 [2].

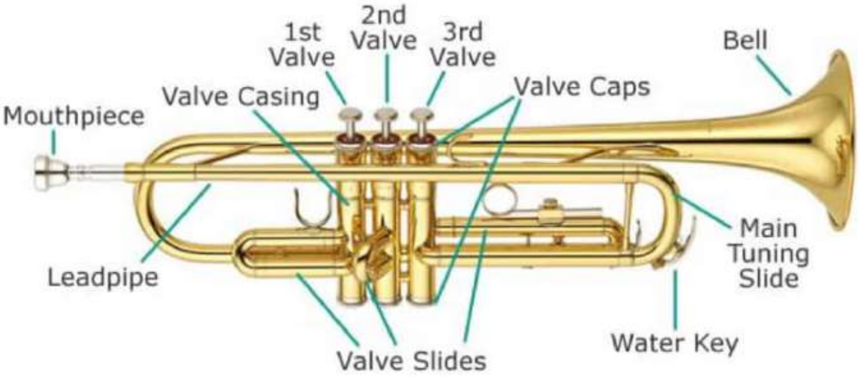


Figure 1. Structure of a standard B-flat trumpet.

In various marching band settings, large temperature differences significantly affect pitch. The effect of temperature on trumpet tuning is a known phenomenon, however, no work has been done to quantify its effect on a trumpet's main tuning slide. A trumpet player typically uses the Tenor C (466.1638 Hz) as a tuning pitch. Reading the tuner, they pull out the main tuning slide if the note is sharp and push it in if the note is flat [3]. Since it is known that pitch varies directly with ambient temperature, trumpet players constantly need to pull out the slide when it is hot and push it in when it is cold [4]. Both ambient temperature and main tuning slide length pulled out alter the frequency of standing sound waves that fit in a trumpet. This project aims to quantify the relationship between them, in order to produce a pitch that is in tune.

In this experiment, whether a pitch was in tune was determined by the difference between the pitch a trumpet produced, measured by a tuner, and the standard frequency in the equal tempered scale. A trumpet produces different pitches, since it contains three valves, giving eight different combinations of fingerings. The different pipe lengths that are created allow standing waves with varying frequencies to fit. Then, the mouthpiece and the bell alter frequencies of standing waves, allowing the entire chromatic scale to be produced [5]. The standard frequencies in which the trumpet's pitches were compared to are based on the equal tempered chromatic scale, which relates consecutive notes on the chromatic scale by a factor of 122 Hz [6].

In this project, to pinpoint the nature of the relationship between ambient temperature and tuning pitch, the temperature distribution across the trumpet at various playing times and ambient temperatures were analyzed in relation to trumpet pitch. After trumpet players obtained an approximation of the ambient temperature, the model obtained through regression analysis suggested necessary adjustments to the main tuning slide length at performances and settings where tuning the trumpet is inconvenient. This work may motivate future research on other brass instruments to investigate if similar relationships exist.

Experimental Methods

Figure 2 shows a schematic of the experimental setup used in this report. To summarize this setup, a standard Bach B-flat trumpet with Bach 3C mouthpiece was obtained. Five HiLetgo Digital LCD Fridge Thermometers with Probe were taped to the respective locations on the trumpet (Figure 2) to measure the temperature. A sixth thermometer was set on the side to measure ambient temperature, with the probe suspended in the air. 3 paper rulers were taped to the tuning slides to measure their lengths. One Korg CA chromatic clip-on tuner was clipped to the bell of the trumpet to measure pitch accuracy. One timer was set on the side to keep track of the playing time during each experimental trial. Experiments were performed in varying temperatures controlled by air conditioning, location, time of the day, and weather.

Figure 2. Experimental Setup used in this study. It consisted of 6 temperature sensors, three paper rulers, a trumpet, tuner, and timer. Five temperature sensors and three paper rulers were taped to the trumpet at respective locations labeled above.

Setup Procedure

1. For calibration, the 6 temperature sensors were submerged in water for 5 minutes. The average temperature displayed was calculated and the deviation of each sensor from the average was noted.
2. Five temperature sensor probes were taped to the trumpet as shown in Figure 2. Each sensor was labeled to consistently place it in the same position for every trial.
3. A sixth temperature sensor was set on the side to display ambient temperature.
4. 3 paper rulers were obtained and taped on the ends of the tuning slides labeled above.
5. The tuner was turned on and clipped to the opening of the bell.

Experimental Trial Procedure

1. The experimental setup was exposed to the ambient temperature for at least 15 minutes, to ensure the temperatures across the trumpet were stable.
2. A timer was set for 30 minutes and started when ready.
3. The temperature displayed on each of the 6 sensors was recorded, to check the overall temperature distribution across the trumpet.
4. A breath was taken and the Tenor C was sustained long enough to observe the tuner reading.

5. The main tuning slide was pulled out when the pitch was sharp, and pushed in when the pitch was flat. Adjustments were made until the tuner indicated that the pitch was in tune.
6. A paper ruler was used to measure the length of the main tuning slide (L) when it was pulled out, accurately to within 1 millimeter, as shown in Figure 3. The value was recorded.
7. The trumpet was warmed up continuously as long tone and articulation exercises were played to establish a stable temperature inside the instrument.
8. The temperature recording, tuning, and warm-up procedures in steps 3-7 were repeated twice, once after playing for 5 minutes, and once after playing for 10 minutes.
9. After 15 minutes, the temperature displayed on each of the 6 sensors was recorded.
10. For every note on the chromatic scale from Low F sharp to Double High C, the tuning procedures in steps 4-6 were repeated, but the first and/or third valve tuning slides were adjusted this time. The total length of all tuning slides pulled out was recorded.
11. After 30 minutes, the temperature displayed on each of the 6 sensors was recorded.
12. Steps 1-11 were repeated for 5 additional trials in different ambient temperatures. All raw data is listed in Table A1-A6 in the Appendix.

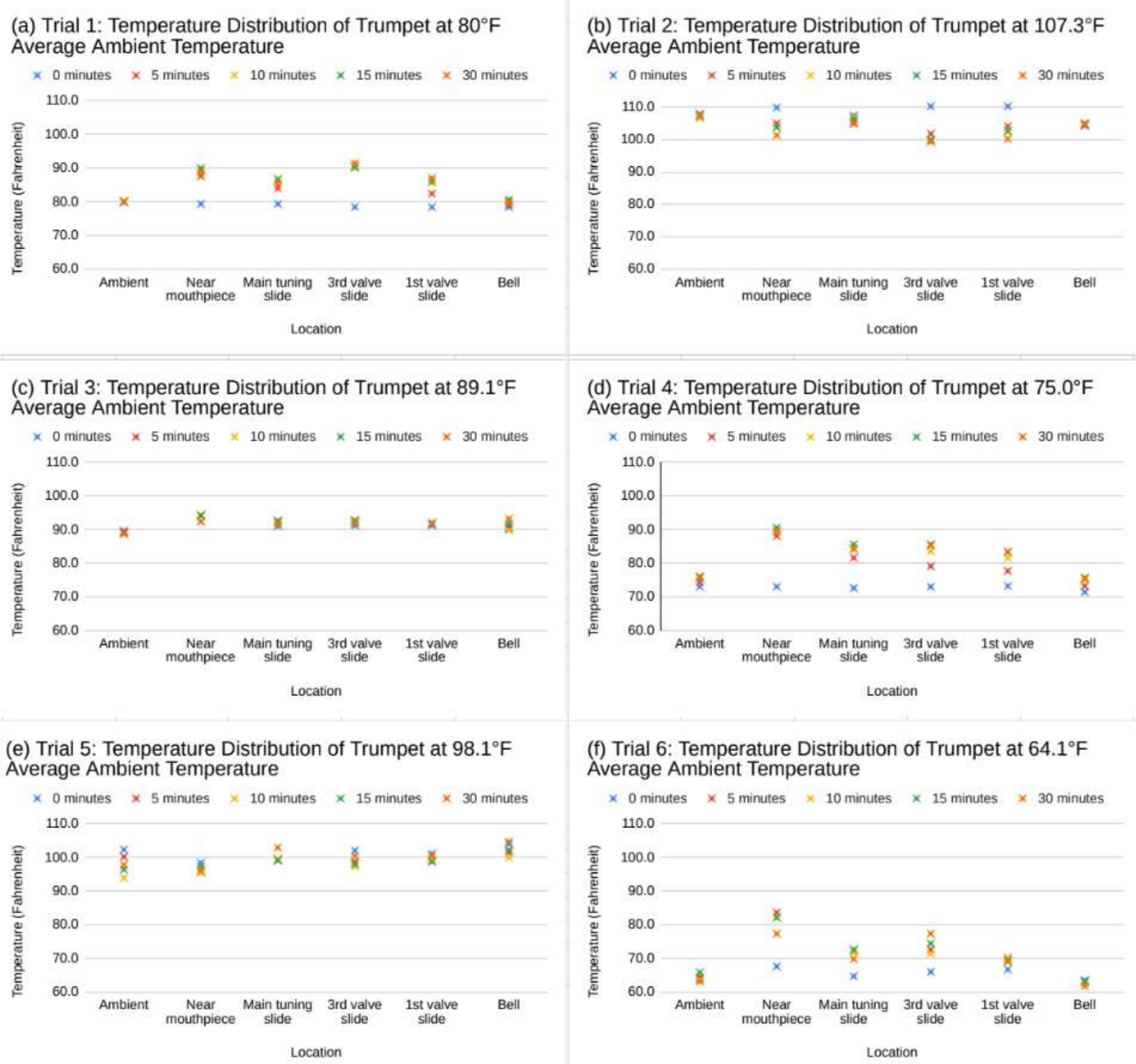
Figure 3. U-shaped main tuning slide of the trumpet. When the slide is pulled out, the additional length of each of the two ends is L/2. The total length pulled out is L, the sum of the two ends of the slide.

Data Analysis
After all six experimental trials were performed, the ambient temperature, temperature distribution across the trumpet, and main tuning slide lengths pulled out, were plotted in different combinations with Google Spreadsheet. The pairwise relationships between several variables were plotted to determine if there were any significant trends. Different forms of regression analysis were performed with Desmos graphing calculator, to describe the trend between ambient temperature and main tuning slide length pulled out. Of the regression analysis that were attempted, the function with the highest R^2 value was chosen for further investigation.

Results and Discussion

In each of the 6 experimental trials, the ambient temperature, temperature distribution across the trumpet, and the main tuning slide length pulled out after 0, 5, 10, and 15 minutes of playing were recorded. Additional temperature values were recorded 30 minutes after playing. With the data collected, the relationship between four sets of quantities were analyzed and described as follows.

Firstly, the temperature distribution across the trumpet was analyzed. In the plots shown in Figure 4a-f, there was not a clear correlation between the temperature a sensor displayed and its location. No trend could be identified that describes how temperature varied with the distance the airstream traveled from the mouthpiece. The temperature at the bell was close to the ambient temperature and remained independent of playing time. Temperatures near the mouthpiece, in the main tuning slide, in the third valve slide, and in the first valve slide, approached body temperature (97°F-99°F) over time. After 5-10 minutes of trumpet playing, these temperatures remained steady.



(a) Trial 1: Temperature Distribution of Trumpet at 80°F Average Ambient Temperature

(b) Trial 2: Temperature Distribution of Trumpet at 107.3°F Average Ambient Temperature

(c) Trial 3: Temperature Distribution of Trumpet at 89.1°F Average Ambient Temperature

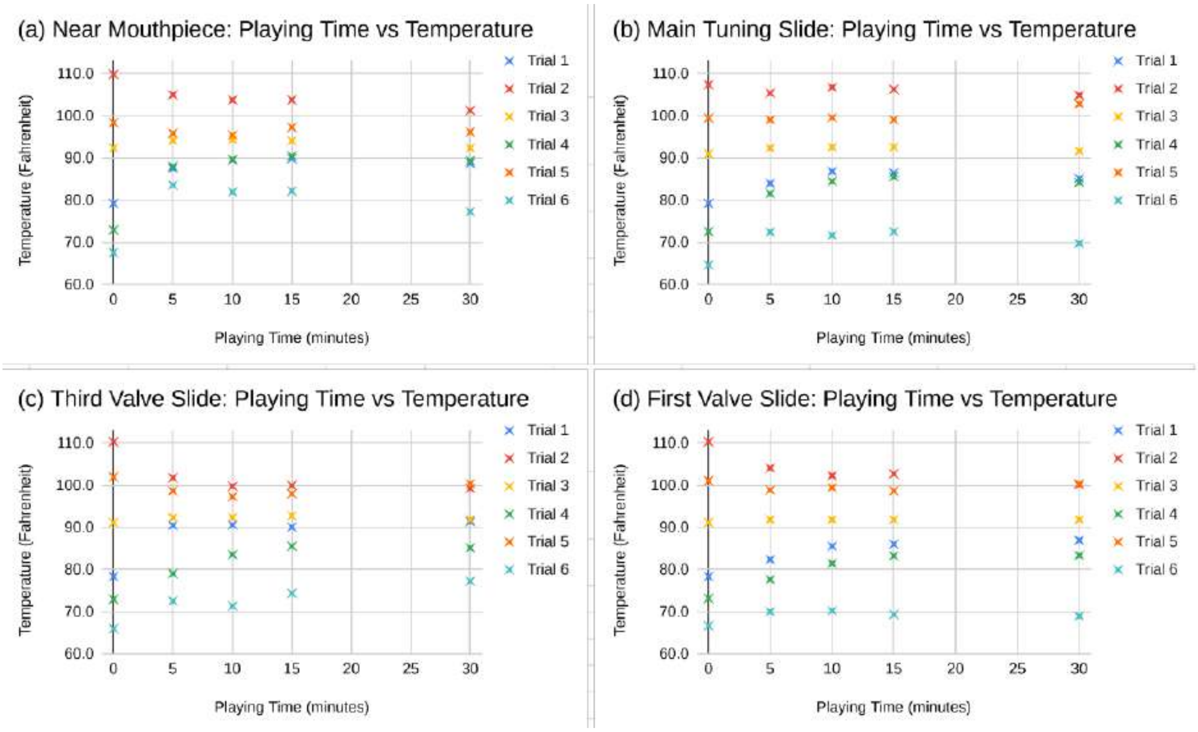
(d) Trial 4: Temperature Distribution of Trumpet at 75.0°F Average Ambient Temperature

(e) Trial 5: Temperature Distribution of Trumpet at 98.1°F Average Ambient Temperature

(f) Trial 6: Temperature Distribution of Trumpet at 64.1°F Average Ambient Temperature

Figure 4. Temperature distribution across the trumpet for each of the six experimental trials. Temperature was measured in the ambient, tubing near the mouthpiece, main tuning slide, third valve slide, first valve slide, and the bell.

Secondly, the relationship of playing time versus temperature at each location inside the trumpet was analyzed. In the plots shown in Figure 5a-d, there was not a clear correlation between initial temperatures and values they approached over time. During trumpet playing, the trend of how temperature changed at each location of the trumpet could not be predicted. Temperatures near the mouthpiece, in the main tuning slide, in the third valve slide, and in the first valve slide were plotted, with playing time on the x-axis, and temperature on the y-axis. Over time, the temperatures across trials decreased in range and got closer to the body temperature (97°F-99°F) over time. They remained steady after 5-10 minutes of playing.



(a) Near Mouthpiece: Playing Time vs Temperature

(b) Main Tuning Slide: Playing Time vs Temperature

(c) Third Valve Slide: Playing Time vs Temperature

(d) First Valve Slide: Playing Time vs Temperature

Figure 5: Plots of playing time versus temperature in (a) tubing near the mouthpiece, (b) main tuning slide, (c) third valve slide, and (d) first valve slide.

Thirdly, the relationship of note pitch versus total tuning slide length pulled out was analyzed. Having tuned the Tenor C and adjusted the main tuning slide length, the notes F#3, C#4, D4, F4, A4, F5, G#5 and A5 usually had a sharp pitch and required additional first or third valve tuning slide lengths. The other notes were either naturally flat (thus no adjustments could be made) or in tune. The harmonic series that explains this phenomenon is outside the scope of this study.

Finally, the relationship of temperature distribution versus main tuning slide length pulled out was analyzed. The main tuning slide length pulled out varied directly with temperature at all six measuring locations. Since the measurement of ambient temperature was most practical to a trumpet player, and it had a clear relationship with the main tuning slide length pulled out, regression analysis was used to precisely quantify this relationship. To determine this relationship, data points collected after playing the trumpet for 10 and 15 minutes were chosen. This choice ensured that the temperature distribution inside the trumpet reached a steady state. Data points were plotted, with ambient temperature in Fahrenheit on the x-axis and main tuning slide length pulled out in centimeters on the y-axis. Then, different regression models were considered, shown in Figure 6a-f.

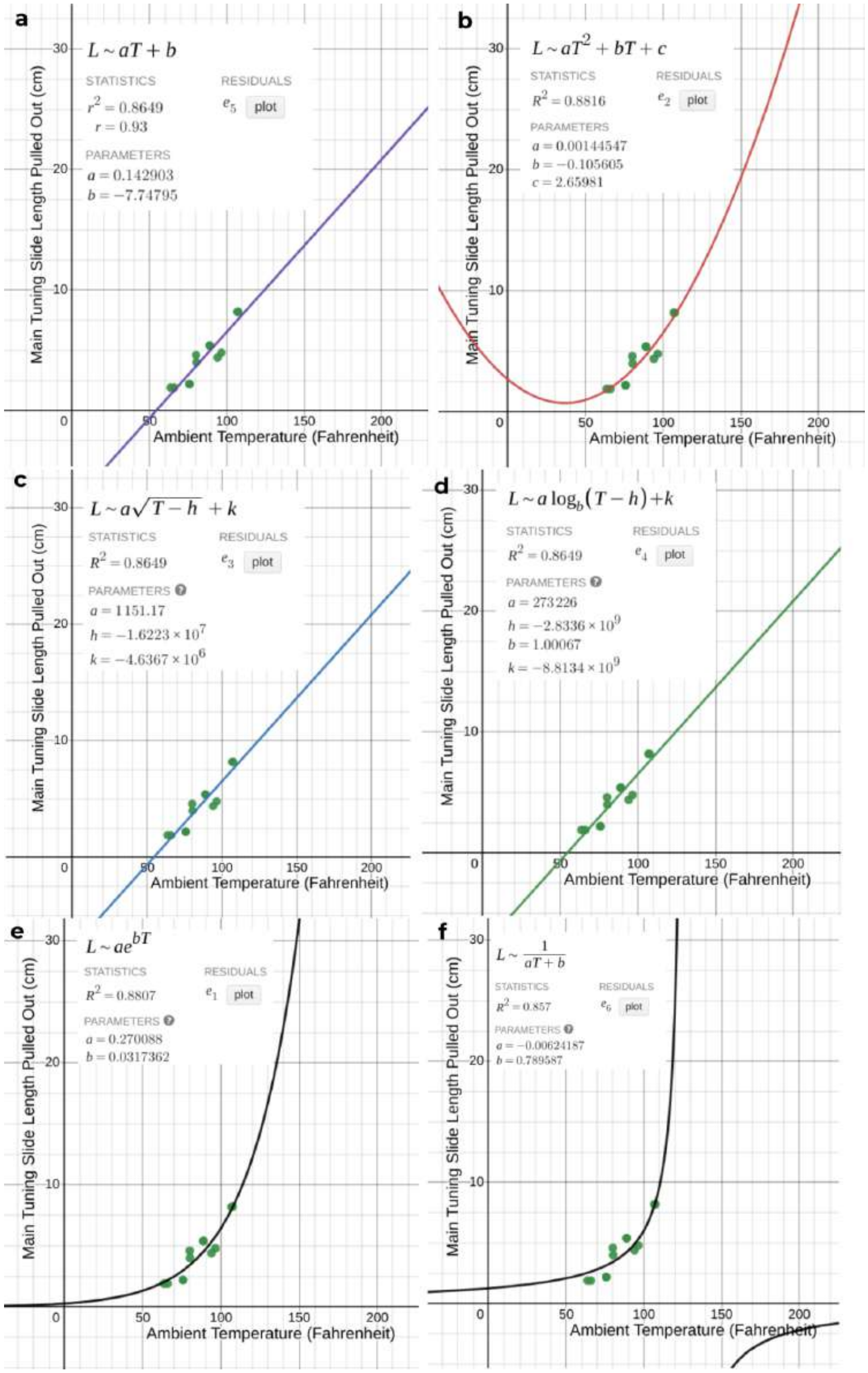


Figure 6. Least squares regression models for ambient temperature versus main tuning slide length pulled out. (a) Linear, (b) quadratic, (c) square root, (d) logarithmic, (e) exponential, and (f) inverse.

Models in Figure 6a, c, and d were ruled out, since the length of a tuning slide could not be negative. The quadratic model in Figure 6b was ruled out, since the main tuning slide length should not be pulled out more, as temperature decreases and approaches zero. The inverse model in Figure 6f was ruled out since there was a vertical asymptote, and the length of the main tuning slide could not be undefined. It also did not make sense that the length values to the right of the asymptote were negative. Considering that the mathematical properties of a model must suit the physical hypothesis that main tuning slide length pulled out varies directly with temperature, a least squares exponential regression model in the form $y = ae^{bx}$ had the highest correlation. It is a continuous function, has a domain of all real numbers, has only positive output values, and is always increasing. Through computation, $a \approx 0.270088$ and $b \approx 0.0317362$. The graph of the model and data points are shown in Figure 7. Error bars were drawn for each data point, representing the ± 2°F error of temperature measurement, and ± 0.05cm error of length measurement. Based on this model, a table of values with ambient temperatures (T) from 30°F to 110°F (-1.1°C to 43.3°C) in 2°F increments and their corresponding additional main tuning slide lengths (L), were generated and shown in Table 1. Since the main tuning slide is a u-shaped tube, as shown in Figure 3, the length (L) computed through the model is the sum of the two ends of the slide. For convenience of measurement, values for (L/2) representing one end of the slide are provided.

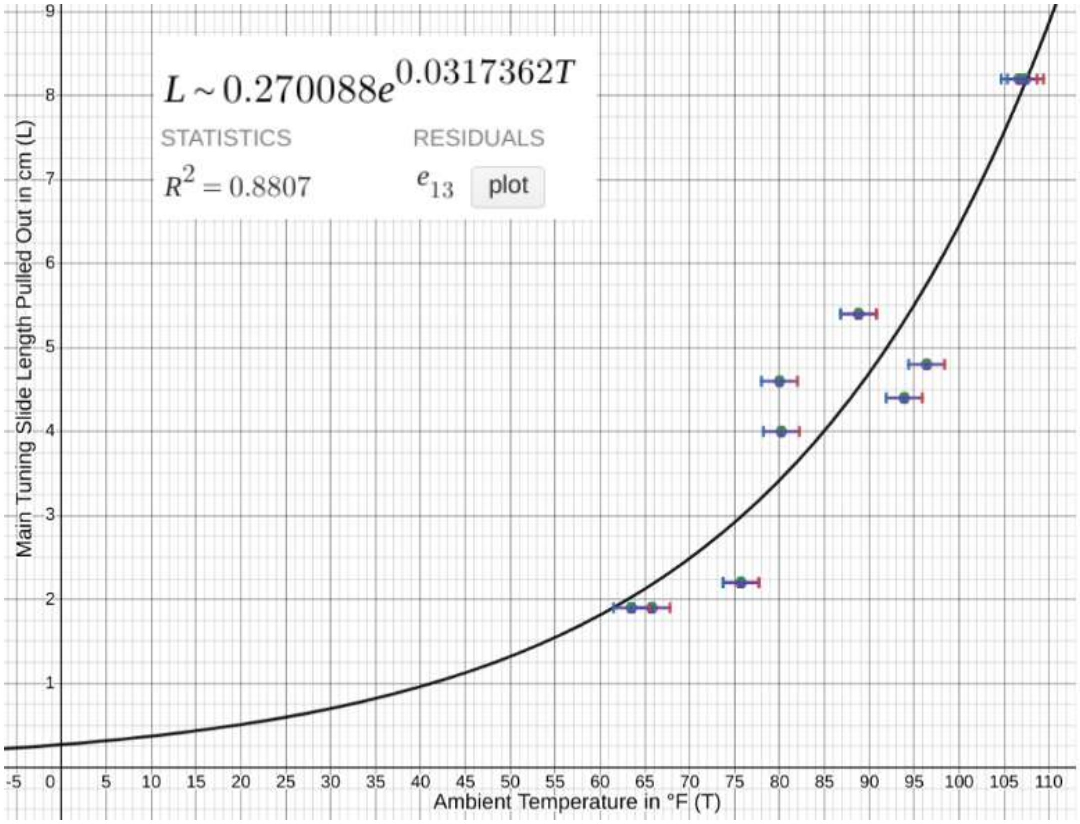


Figure 7. Exponential least squares model for the relationship between ambient temperature (T) in Fahrenheit and main tuning slide length pulled out (L) in centimeters, given by $L = 0.270088e^{0.0317362T}$. Data points and their corresponding error bars were plotted.

Table 1. Table of values generated from $L = 0.270088e^{0.0317362T}$, with ambient temperatures (T) ranging from 30°F to 110°F in 2°F increments, and their corresponding values of main tuning slide length pulled out (L) and length of one end of the slide pulled out (L/2).

Quantities	Ambient temperature (T)		Main tuning slide length to pull out (L)		Half of main tuning slide length to pull out (L/2)	
Units	°F	°C	cm	in	cm	in
Values	30.0	-1.1	0.70	0.28	0.35	0.14
	32.0	0.0	0.75	0.29	0.37	0.15
	34.0	1.1	0.79	0.31	0.40	0.16
	36.0	2.2	0.85	0.33	0.42	0.17
	38.0	3.3	0.90	0.36	0.45	0.18
	40.0	4.4	0.96	0.38	0.48	0.19
	42.0	5.6	1.02	0.40	0.51	0.20
	44.0	6.7	1.09	0.43	0.55	0.21
	46.0	7.8	1.16	0.46	0.58	0.23
	48.0	8.9	1.24	0.49	0.62	0.24
	50.0	10.0	1.32	0.52	0.66	0.26
	52.0	11.1	1.41	0.55	0.70	0.28
	54.0	12.2	1.50	0.59	0.75	0.30
	56.0	13.3	1.60	0.63	0.80	0.31
	58.0	14.4	1.70	0.67	0.85	0.34
	60.0	15.6	1.81	0.71	0.91	0.36
	62.0	16.7	1.93	0.76	0.97	0.38
	64.0	17.8	2.06	0.81	1.03	0.41
	66.0	18.9	2.19	0.86	1.10	0.43
	68.0	20.0	2.34	0.92	1.17	0.46
	70.0	21.1	2.49	0.98	1.25	0.49
	72.0	22.2	2.65	1.04	1.33	0.52
	74.0	23.3	2.83	1.11	1.41	0.56
	76.0	24.4	3.01	1.19	1.51	0.59
	78.0	25.6	3.21	1.26	1.61	0.63
	80.0	26.7	3.42	1.35	1.71	0.67

	82.0	27.8	3.65	1.44	1.82	0.72
	84.0	28.9	3.88	1.53	1.94	0.76
	86.0	30.0	4.14	1.63	2.07	0.81
	88.0	31.1	4.41	1.74	2.20	0.87
	90.0	32.2	4.70	1.85	2.35	0.92
	92.0	33.3	5.01	1.97	2.50	0.99
	94.0	34.4	5.33	2.10	2.67	1.05
	96.0	35.6	5.68	2.24	2.84	1.12
	98.0	36.7	6.06	2.38	3.03	1.19
	100.0	37.8	6.45	2.54	3.23	1.27
	102.0	38.9	6.88	2.71	3.44	1.35
	104.0	40.0	7.33	2.88	3.66	1.44
	106.0	41.1	7.81	3.07	3.90	1.54
	108.0	42.2	8.32	3.28	4.16	1.64
	110.0	43.3	8.86	3.49	4.43	1.74

Such a relationship exists, since as ambient temperature increases, the frequency of sound waves increases [7]. Pulling out the main tuning slide increases the total pipe length of the trumpet, in order to allow standing sound waves with longer wavelengths and lower frequencies to fit in. Conversely, when ambient temperature decreases, the frequency of sound waves decreases, and the main tuning slide should be pushed in.

A potential source of error in these experiments was the pitch inaccuracy produced by the embouchure while a note was sustained. If the airstream blown through the mouthpiece was not constant, both the sound intensity and pitch would fluctuate. If there was excessive tension in the lips that inhibited a resonant sound to be produced, the resulting pitch would deviate from the frequency of a standing wave. If pitch was bent down, the tuner would indicate that it had a lower pitch, therefore decreasing the tuning slide length pulled out. Conversely, if pitch was bent up, the tuning slide length pulled out would increase.

To validate the mathematical relationship developed herein, a trumpet player can obtain an approximation of ambient temperature from a temperature sensor probe or thermostat at a location with steady temperature. A trumpet player can choose locations with lower temperatures, to verify the trend of this relationship. Then, at the same location, they can warm-up continuously on the trumpet for 5-10 minutes. Afterwards, they can tune the Tenor C with a clip-on tuner and adjust the main tuning slide length. Finally, they can measure the length pulled out, and compare its difference with the predicted value of the model, at the measured ambient temperature. Using this relationship, trumpet players can make adjustments to the trumpet before tuning, and prioritize intonation during the tuning process.

Conclusions

On a standard B-flat trumpet, to play a Tenor C (466.1638 Hz) in tune, the approximate mathematical relationship between ambient temperature (T) and the main tuning slide length pulled out (L) was found to be $L = 0.270088e^{0.0317362T}$, and Table 1 was generated with this model for the efficiency of a trumpet player's tuning process. With this model, even if trumpet players differ in playing technique, they can be assured that a similar exponential trend can be found to describe their trumpet playing because the underlying physics is the same. After adjusting the main tuning slide using the values in Table 1, only minimal additional adjustments might be necessary. This model facilitates better intonation, equipping trumpet players to attentively listen to their own pitch, and adjusting their embouchure and air flow. This is especially important during ensemble rehearsals and performances in outdoor settings where intonation largely improves sound quality. The following procedure is suggested to take advantage of the findings of this report:

1. Warm-up the trumpet for 5-10 minutes to establish a steady temperature inside the instrument.
2. Obtain an approximation of the ambient temperature T.
3. On the table, in the row of the ambient temperature value closest to T, look up the L/2 value.
4. Using a measuring device or drawn markings, pull out the main tuning slide so that each end is pulled out by a length of L/2.
5. Attach a tuner on the bell of the trumpet, and sustain the Tenor C note. The tuner should indicate that the pitch is close to being in tune.
6. Make additional adjustments as needed. As long as the ambient temperature stays steady and the trumpet is constantly being played on, the length of the main tuning slide will produce an accurate pitch.

The relationship developed in this report may be presumably applicable to other forms of brass instruments because they are all made of the same metal. The specific relationship parameters may differ for different instrument designs, but all brass metal will respond the same to temperature. Thus, this work may inspire other brass instrument players, who tune by adjusting tuning slide lengths, to develop similar relationships for their instruments.

Limitations

Without access to a controlled temperature chamber with a wide range of temperatures, the performed experiments were limited to ambient temperatures controlled by air conditioning, location, time of the day, and weather. In addition, without access to different trumpet players, the experiments were performed and data was collected from a single trumpet player.

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