



Empirical Application of Rubinstein Type Bargaining Models Among High School Students

Andrew Cheng

Abstract:

This paper describes results of a psychological research experiment on High School students. The Rubinstein game is a theoretical game where two players go back and forth to determine how to split a reward that is continuously decreasing in size. Although the Rubinstein game has been extensively studied and researched, very little has been done about alterations, especially when turning it into a communicative game. I conducted separate tests with three different trial groups. In the first test, participants were given the standard Rubinstein game to play. In the second and third, participants were given alterations of the game, in which the second had a small reward if they could not reach an agreement by the end of the game and in the third participants were allowed limited online communication before starting the game. Similar to Güth's findings, participants generally had a more even allocation of money in all trials than predicted, in which the communication group was most even (Güth 1982). Therefore, communication has a bigger impact on initial split than outside options. However, there were limitations to this study, and future research is needed to determine the impact of factors such as average income, age, and location of study.

Keywords: Game Theory, Rubinstein Model, Negotiation, Economics, Psychology, Statistics

Introduction

The Rubinstein bargaining model is a well established and studied experiment in the world of game theory, where two players take turns proposing how to divide a single prize, in this case a pot of money (Rubinstein 1982). One player proposes a split, and the other can choose to either accept or reject. If rejected, the roles switch places and the total pot is reduced by a set number. Fully researched, this game has a subgame-perfect equilibrium in which it is ideal to reach an agreement in the first round with the split of a game with 3 rounds and \$5 pot being exactly \$3.75 for the proposer and \$1.25 for the receiver. However, further research from others has shown that shared knowledge, among other factors, play a role in pushing agreements towards more equal splits (Roth and Malouf 1979). This topic goes back and forth, as researchers such as Binmore show that by modifying things such as roles and number of games, human behavior shifts more towards theoretic predictions (Binmore, Shaked and Sutton 1984)

However, research in this area has run the game uncommunicatively, eliminating potential factors that could influence the outcome (Neelin, Sonnenschein, and Spiegel 1986). Yet, human interaction plays a major role in our daily lives, potentially heavily swaying how one perceives their opponent in a non-cooperative game. For example, one study found experimental evidence that identification of one's opponent improves solidarity in the prisoner's dilemma game (Bohnet and Frey 1999). Therefore, this paper will have some focus on human interaction, while also testing the main game for control and a small alteration as well. In this study, participants were split into three groups or game types, all of which having a max pot of \$5 and a reduction of 50% per round: the original Rubinstein game, an outside option game where a pot of \$0.25 is given to the original receiver if no proposal is accepted throughout, and the aforementioned communication game where before the proposals started the players would anonymously interact.

The hypothesis is the following: for the communication game, I believe that including time before the game for the two players to talk would end up decreasing the difference between the two allocations. This is because the receiver will most likely realise that they are going to gain less money than the offeror, and will attempt to bargain for a more equal split. Additionally, this may humanize the opponent more, causing people to favor a more equal split as well. For the outside option, I believe that giving the receiver an additional option will also end up making the starting deal closer to an equal split. This is because the backwards induction shows that the original offer should result in a more favorable split for the receiver (Table 1) :

Table 1: Backwards Induction of Outside Offer Rubinstein game

Round	Pot	Original Offeror	Original Receiver
3	1.25 (0.25 if rejected)	1.00	0.25
2	2.5	1.00	1.5
1	5	3.5	1.5



Methodology

Game Design and Data Collection:

In this study, I gathered data on a total of 60 students, ranging from grades 10-12 to participate online. Each was randomly assigned to the classic, communication, or outside offer group and given a random role as either receiver or offeror, and were paired with another of the same group. They were then given the set instructions depending on the role and group, and any questions were answered using the instructions (see Appendix). They were then to submit their answers to me, in which I would then transmit it to the opposing player.

The game went as follows: if in the communication group, the players would have 30 seconds in a call or a total of 4 messages with a 30-word limit for each message before any splitting of the pot would occur, though all participants chose to message each other since calling was too difficult to coordinate. The receiver would be the first one to send the message, as they had the mathematical disadvantage. Once the messaging finished the game would start, the offerer would send the split, and the receiver would choose to accept or decline the offer. Once the offer was accepted, the game would end. If denied, the roles would swap, and the game would go to the next round with the total pot halving in amount. If the third pot of \$1.25 was denied, both players would get nothing in all groups except for the outside option, where the original receiver would get \$0.25.

Data Analysis

I collected the data by inputting each response into a Google spreadsheet. Once the data was collected, I trimmed the data, getting rid of one outlier from both the communication and outside offer group as outliers can bias the results of the statistical tests, though both data sets were analyzed to compare the effect of the trimming. Additionally, the analysis performed on data focused on the primary offer, leading to the possibility of outliers.

I used two methods to analyze both the raw and the trimmed data: the Welch's one way ANOVA test and a regression model. I looked at the average differences in the offer receiver split in each group. Then, using the Welch's ANOVA, I examined if the group means among the groups are the same in the population. I used the Welch's version to produce a valid test statistic because of unequal variances across the groups (West 2021). Because group mean pairs cannot be examined using ANOVA, the Ordinary Least Squares (OLS) regression model was performed to investigate the difference in group means between pairs. The dependent variable is the difference in the split in the first round between the offeror and receiver, and the independent variable is the group identity with the reference group as the Classic group. Thus, each group coefficient in the model provides the average difference between the Classic group's mean and its own group's mean.

Results

Descriptive Results

With a total of 60 participants and having three groups each with two players, a total of five games per group were played. Out of the total of fifteen games played, only two had the receiver deny the first offer, in which the games lasted a total of three rounds ending in denying the final round. Within the data sets of the three groups, the communication and outside offer each had an outlier, while the classic did not. The group means of the difference in the split of money for the classic was 0.954 with the standard deviation being 0.860. Including the outliers, the group means of the communication and outside were 0.670 and 0.666, and the standard deviations were 1.609 and 1.472 respectively. Ignoring the outliers, the means dropped to 0.189 and 0.240 while the standard deviation was 0.750 and 0.771 respectively.

Statistical Test Results

A Welch's one way ANOVA was then done on the trimmed data (Table 2), with an F-statistic of 2.40, numerator degrees of freedom of 2, denominator degrees of freedom of 16.62, and a p-value of 0.12. This was an improvement compared to the raw data (F-stat = 0.19, p-value = 0.82), but it is still not statistically significant at the $\alpha = 0.05$ or even the $\alpha = 0.1$ level, failing to reject the null hypothesis. This means we cannot conclude that the group means in the population are truly different.

Table 2: Welch's ANOVA Test Results on Raw and Trimmed Data

Data	F	DF	Denominator Df	Pvalue
Raw	0.19	2	16.28	0.83
Trimmed	2.4	2	16.62	0.12

Note: Raw Data consists of 30 observations across 3 groups; Trimmed data consists of 28 observations across 3 groups after removing two outliers.

After this, a regression analysis was done with the trimmed data (Table 3). The model predicted that the communication group would have a group mean that is \$0.77 less than the classic group, assuming all else fixed (-0.77, p-value = 0.055). Similarly, The model predicted that the outside group would have a group mean that is \$0.71 less than the classic group, assuming all else fixed (-0.71, p-value = 0.072). The p-value of the communication group was 0.055, while for the outside group it was 0.072. Given the $\alpha = 0.05$ confidence level, we cannot reject the null hypotheses that either the communication group mean or the outside group mean is different from the classic group mean, though the communication group was very close to being statistically significant. However, both are statistically significant at the $\alpha = 0.1$ level.

Table 3: Ordinary Least Squares Regression Coefficient Estimates

Coefficient	Estimate	Pvalue
Intercept	0.9540	0.00121
Communication	-0.7651	0.05473
Outside	-0.7140	0.07169

Note: Performed on trimmed data; Dependent Variable is the difference in offeror/receiver split in the first round. Independent variable is the group identity (Classic, Communication, Outside). The Classic group is the reference group.

Discussion

In each statistical test, the p-values show that the change was not statistically significant at the $\alpha = 0.05$ level, meaning that we could not conclude a difference in the population means among the three groups. However, from the regression analysis, the results show that the communication group had a coefficient estimate farther than zero than the outside offer group having a p-value just slightly above 0.05 whereas the other was at 0.07. Additionally, in every single group, classic included, the average split of the money was more centralised towards 50/50 than the ideal mathematical solution for the situation. Still, the proposed hypothesis was semi correct, as the average initial offer dropped for both experimental groups and the average split was closer to even than the control group. This study is most likely not a reflection on the average human. It was done with only a sample size of five games per group, including two of the fifteen being outliers. Additionally, the data analysis was done on the first offer only, so the possibility of someone rejecting a decent offer and it not being noted is possible and happened in one game. There is also a possibility of the region playing a factor, as the area is a more well off one, causing some participants to not feel the need of maximizing their profit.

Conclusion

This study examined how communication and an outside offer impacted the proposal decisions of participants compared to the original Rubinstein game among high school students. The primary objective was to explore how communication affected how participants played a non-cooperative game. The findings indicate that although close, neither game ultimately is able to reject the null hypothesis that on average, the offeror's payoff is the same among all three groups. This study was a very small preview of what could be larger research into how not only communication, but also other factors like age and income come into play when one makes decisions. This study has shown that although seemingly different, the impact of these factors do not make enough impact to clearly and statistically show a difference in the decisions made. Still, this research acknowledges limitations such as a small and concentrated sample size. Additionally, the analysis largely focused on the first proposal, thus not fully taking into account games that went past the first round. Future research directions should focus on isolating individual factors and increasing sample size to reduce uncertainty of the extent to which each factor plays in affecting one's decision making.

Funding Sources

The author declares that there were no funding sources related to this work.



Conflicts of Interest

The author declares that there are no conflicts of interests related to this work.



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Appendix

All participants were sent the introduction instructions no matter the game. They were also sent the specific game and role instructions based on which they were assigned. The three games they could be assigned to were the base, communication, and outside offer Rubinstein game.

Instructions for all Rubinstein games:

Introduction: You will be playing a game called the Rubinstein game, where you and one other unknown player will be deciding how to split a pot of money. Each player is assigned one of two roles: the offeror or the receiver. Both parties are told the initial amount of the pot, and the offeror gets to decide how to split that pot. Then, the receiver can either accept or decline the deal. If they accept, the game will be over and each player will receive their designated payout. However, if they decline, the game will move to the next round, where the roles are swapped and the total pot decreases by some amount.

Base Rubinstein game: offeror

Game: Your starting role is the offeror. In this specific game, the total pot starts at \$5. There is a maximum of 3 rounds, with the end of each round decreasing the total pot by 50% if the receiver declines the deal. You can decide to split the pot between you and the receiver in any way you want, as long as the sum adds up to the total pot, and message it to me. I will then let you know if the receiver accepted it—in which case you win the share of the pot you proposed, or declined it—in which case I will tell you the split of the (now reduced) pot that the other player is offering.

Base Rubinstein game: receiver

Game: Your starting role is the receiver. In this specific game, the total pot starts at \$5. There is a maximum of 3 rounds, with the end of each round decreasing the total pot by 50% if the receiver declines the deal. The offeror will decide to split the pot between them and you in any way you want, as long as the sum adds up to the total pot, and message it to me. I will then let you know the offer, and you will be able to accept it—in which case you win the share of the pot that was proposed, or decline it—in which case you will create a new split with the reduced pot that the other player will accept or decline.

Communication Altered Rubinstein game: offeror

Game: Your starting role is the offeror. In this specific game, the total pot starts at \$5. There is a maximum of 3 rounds, with the end of each round decreasing the total pot by 50% if the receiver declines the deal. You can decide to split the pot between you and the receiver in any way you want, as long as the sum adds up to the total pot, and message it to me. I will then let you know if the receiver accepted it—in which case you win the share of the pot you proposed, or declined it—in which case I will tell you the split of the (now reduced) pot that the other player is offering. However, in this game, you will have the chance to communicate with the other player before the game starts, with either 30 seconds in a call or a total of 4 messages with a 30-word limit for each message.

Communication Altered Rubinstein game: receiver

Game: Your starting role is the receiver. In this specific game, the total pot starts at \$5. There is a maximum of 3 rounds, with the end of each round decreasing the total pot by 50% if



the receiver declines the deal. The offeror will decide to split the pot between them and you in any way you want, as long as the sum adds up to the total pot, and message it to me. I will then let you know the offer, and you will be able to accept it—in which case you win the share of the pot that was proposed, or decline it—in which case you will create a new split with the reduced pot that the other player will accept or decline. However, in this game, you will have the chance to communicate with the other player before the game starts, with either 30 seconds in a call or a total of 4 messages with a 30 word limit for each message.

Outside offer Altered Rubinstein game: offeror

Game: Your starting role is the offeror. In this specific game, the total pot starts at \$5. There is a maximum of 3 rounds, with the end of each round decreasing the total pot by 50% if the receiver declines the deal. If the game goes to the final round and the receiver decides not to accept the deal, the receiver will receive an outside \$0.25 pot and the offeror will receive nothing. You can decide to split the pot between you and the receiver in any way you want, as long as the sum adds up to the total pot, and message it to me. I will then let you know if the receiver accepted it—in which case you win the share of the pot you proposed, or declined it—in which case I will tell you the split of the (now reduced) pot that the other player is offering.

Outside offer Altered Rubinstein game: receiver

Game: Your starting role is the receiver. In this specific game, the total pot starts at \$5. There is a maximum of 3 rounds, with the end of each round decreasing the total pot by 50% if the receiver declines the deal. If the game goes to the final round and the receiver decides not to accept the deal, the receiver will receive an outside \$0.25 pot and the offeror will receive nothing. The offeror will decide to split the pot between them and you in any way you want, as long as the sum adds up to the total pot, and message it to me. I will then let you know the offer, and you will be able to accept it—in which case you win the share of the pot that was proposed, or decline it—in which case you will create a new split with the reduced pot that the other player will accept or decline.