Tests of Special Relativity

Alex Xue

Abstract

The invariance of the speed of light is a core principle of special relativity. There were multiple experiments that led to this theory. This paper compiles the existing research on this topic into a chronological framework, while also clearly showing all the calculations of each experiment by phrasing the calculations through the Mansouri-Sexl test theory. Unifying these experiments under one test theory allows the reader to quantitatively compare improvements that resulted from changing interferometry to optical resonators. This paper discusses the Michelson-Morley experiment, Kennedy-Thorndike Experiment, and a modern recreation of the KT experiment, “Tests of Relativity Using a Cryogenic Optical Resonator”.

I. Introduction

In the 1700s, it was believed that light took the form of tiny, hard, and indivisible corpuscles (1). However, in 1801, Thomas Young made a critical discovery in optical physics. In his famous Double Slit Experiment, he found out that light did not take the form of the ones described by people like Newton, but instead had wave-like properties. He did this by taking a strong source of light like a laser, and then passing the light through two parallel slits. A screen behind the slits detected and observed the light. If light really took the form of corpuscles, then the screen would observe two bright regions corresponding to where the slits were. However, instead of seeing that, Young saw an interference pattern of bands of bright and dark regions. This could only occur if light had wave-like properties. The waves coming out of the two slits would interfere creating constructive and destructive interference patterns. These “fringes” were exactly what Young observed. Any previous notions that light was not wave-like after this, like Newton’s corpuscles, were completely destroyed (2).

However, at the time physicists believed that waves need something to propagate through. Ocean waves need water to move. Sound waves need a fluid like air to move. Waves when plucking the strings of a guitar need the existence of the string in order to move. So, scientists thought that light should be no different. If light had wave-like properties there must be some medium through which it can be propagated. This medium was referred to as the luminiferous aether (3). The aether was thought to be stationary. However, like moving through any fluid, objects should experience a drag force. This was referred to as the aether wind. We can parameterize the experiments that were conducted to test for the existence of the aether using a test theory. A test theory is very useful because it gives experimenters a clear mathematical equation to test for. In a modern recreation of the KT experiment, the researchers use the following test theory developed by Reza Mansouri and Roman Ulrich Sexl to conceptualize the aether theory (4):

$$c(v,\theta)/c_0 = 1 + A(v^2/c_0^2) + B(v^2/c_0^2)sin^2\theta \qquad (1)$$

If the aether exists, that means that the motion of Earth through space should create a drag force that affects everything on Earth including light. If the aether exists that also means that the speed of an observer should affect the observed speed of light. Therefore, if the experimenters of the Michelson-Morley, Kennedy-Thorndike, and recreations of these experiments can prove that $A = 0$ and $B = 0$, then the aether theory can be proven to be false.

The intent of this paper is not to discover anything groundbreaking, but instead to piece together the experiments over the past century or so into one concise paper. Current literature gives plenty of information about each experiment, but there is a comparatively lack of sources that explain how all of them fit together while maintaining scientific rigor. Furthermore, sources that attempt to do this often use vague analogies or assume a complex technical background that in reality isn't needed. This paper aims to discuss the Michelson-Morley and Kennedy-Thorndike Experiments using only basic algebra (which is all that is necessary) while showing how original and modern versions of both experiments can be unified by the Mansouri–Sexl test-theory. This paper will also include explicit calculations showing how measurements taken from various experiments leads to an expansion of knowledge about the speed of light. These details are often left out in papers.

II. Michelson-Morley Experiment

The Michelson-Morley Experiment tried to detect the hypothetical aether wind. Since the aether was thought to be stationary, but the movement of the Earth caused a drag force, it was hypothesized that light should move at different speeds based on the direction it is going in. In 1887, American physicists Albert Michelson and Edward Morley used an instrument called an interferometer to detect this difference in the speed of light (5).

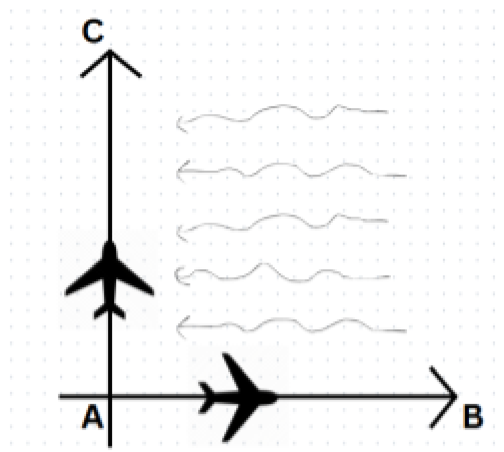

Figure 1

The structure used in the Michelson-Morley Experiment was a set of perpendicular beams with mirrors on the farthest ends, a strong light source and a detector at the other ends, and a beam splitter in the center. The light source would shoot a beam of light at the beam splitter which would send the light in two different directions along the perpendicular paths. The mirrors at the end would reflect the light back to the beam splitter and then to the detector. If the aether medium existed the physicists should have been able to detect a difference in time at which the light beams reach the detector.

While the Mansouri-Sexl test theory can be used to give a quantitative interpretation of the Michelson-Morley experiment, it is easier to think of an analogy to understand how the ideas were developed because they can help us visualize the concepts without getting lost in the maths. We can think of a plane moving with a speed of *c* from points *A* to *B* whose distance is *D*. Imagine a wind blowing with a speed of *v* from *B* to *A*. When the plane is moving parallel to the wind, it is either moving with or against the wind. When the plane moves from points B to A, we find that the total time is $t = D/(c - v)$, where $c + v$ is the rate. When the plane moves from points A to B, that is against the wind, we find the total time is $t = D/(c - v)$ where $c - v$ is the rate. Therefore, we find that the total round trip time of the plane is $(2Dc)/(c^2 - v^2)$. Now, let's imagine that there is no wind and $v = 0$. Then, the total round trip time is $(2D)/c$. Using these two equations, we can find the proportion of the round trip time in moving air to the round trip time in still air:

$$((2Dc)/(c^2 - v^2)) / ((2D)/c) = 1/(1 - (v^2/c^2)) \quad (2)$$

Therefore, the time it takes to do a round trip between A and B when $v > 0$, is greater than the time when $v = 0$ by a factor of $1/(1 - (v^2/c^2))$.

Now, let us imagine that the plane is moving perpendicular to the wind from points A to C. AC has the same distance D as AB. When $v = 0$, the time is still $(2D)/c$. When $v > 0$, the plane must angle itself slightly to the right to ensure it reaches point C. From points A to C and back, the plane has velocity $c\,m/s$, but has a horizontal component of v and a vertical component of x. v is just the component that counters the wind and x can be solved using the Pythagorean Theorem which gives us $x = \sqrt{(c\hat{}2 - v\hat{}2)}$. Therefore, the round trip time is $(2D)/(\sqrt{(c\hat{}2 - v\hat{}2)})$. Using these two expressions we find that the proportion between the round trip time in moving air to still air is

$$1/(1 - (v^2/c^2))^{1/2} \quad (3)$$

Therefore, the time it takes to do a roundtrip from A to C when $v > 0$ is greater than when $v = 0$ by a factor of $1/(1 - (v^2/c^2))^{1/2}$.

Using these expressions, we can find the time difference between the parallel and perpendicular paths. Assuming the wind is still moving from points B to A, we have that the round trip time from A to B is $t = (1/(1 - (v^2/c^2))) \times (2D/c)$, and the round trip time from A to C is $t = (1/(1 - (v^2/c^2))^{1/2}) \times (2D/c)$. We need to subtract the two equations to find the time difference formula. We can use the first two terms of the binomial theorem, $(1 + z)^2 = 1 + nz$, to simplify things. This is an approximation and only holds true when $|z| << 1$ and $v << c$ because the planes will eventually represent the speed of light.

For simplicity sake, let $2D = L$:

$$\begin{aligned}\Delta t &= (1/(1 - (v^2/c^2)))\times (L/c) - 1/(1 - (v^2/c^2))^{1/2} \times (L/c)\\ &= (1 - (v^2/c^2))^{-1} \times (L/c) - (1 - (v^2/c^2))^{-1/2} \times (L/c)\\ &= (L/c)((1 - (-1)(v^2/c^2)) - (1 - (-½)(v^2/c^2)))\\ &= (L/2)((½) \times (v^2/c^2)) = (L/2c)(v/c)^2\end{aligned}$$

We can now use this final equation in the Michelson-Morley Experiment. The experiment was set up such that there were two perpendicular beams whose round trip lengths were 22 meters. While the hypothetical aether is stationary, the movement of the Earth around the Sun causes an aether drag of 3.0×10^4 m/s. Our "planes" are going to be replaced by light which propagates at around 3.0×10^8 m/s.

Substituting the values into our formula yields the equation
$\Delta t = (22/(2 \times 3 \times 10^{8}))((3 \times 10^{4})/(3 \times 10^{8}))^{2} = 3.67 \times 10^{-16}$ seconds.
This number is very small and even the strongest of instruments today would have difficulty detecting this difference. Because of this, a different method needs to be used to measure the time difference. Michelson and Morley used the fringes that were created when the light beams, which take different amounts of time to complete their journey, overlapped (6). From this, a clear pattern of alternating black and white stripes can be seen. This pattern is specific to the wavelength of the light and exactly how the waves overlap. Since the same wavelength of light was used for both beams, if the pattern changed as the experiment was repeated throughout the year, it could only be due to one of the waves being slower than the other. This would provide evidence that the aether medium did exist. They calculated that the drag force of moving through the aether should cause a fringe shift of about 0.40 of a fringe (5). However, when the physicists looked through their interferometer, they saw that the maximum displacement of fringes was 0.02, and the average displacement was far less than 0.01. Since the predicted shift and the observed shift were so different, it was concluded that the aether could not exist.

III. Analysis using the Mansouri-Sexl test theory

We can use an equation that is derived from the Mansouri-Sexl test theory to calculate the fringe shift. This will allow us to compare this experiment to modern recreations.

$$N = (L) \times (f) \times B \times (v^{2}/c^{2}) \times (c/(c(v, 90) \times c(v, 0)) \qquad (4)$$

In this equation, N is the fringe shift, L is the length of the apparatus, f is the frequency of the monochromatic light, B is taken from the test theory, c is the speed of light, and v is the velocity of the observer which in this case is the Earth. The monochromatic light that was used in this

Table 1	
Possible Fringe Shifts	Upper bound of $\|B\|$
0.40	1.08
0.02	0.054
0.001	0.0027
0.0001	0.00027

experiment was a yellow sodium flame which has a frequency of 5.09×10^{14} (5). Using this formula, we can take several hypothetical fringe shifts, as seen in Table 1, and calculate the upper bound of $|B|$. The upper bound of $|B|$ can be calculated using the upper bound of $c(v, \theta)$ which is $c + v$ for all values of θ. Thus we can replace $c(v, 0)$ and $c(v, 90)$ with $c + v$.

IV. Kennedy-Thorndike Experiment

While the Michelson-Morley Experiment showed that light maintains the same velocity no matter which direction it is going in, the Kennedy-Thorndike Experiment wanted to observe the speed of light while the observer was moving at different speeds. Is the observed speed of light dependent on the speed of the observer? The experiment was first conducted in 1932 and the experimenters, Roy J. Kennedy and Edward M. Thorndike wanted to test a core principle of special relativity. Particularly, the constant speed of light for all observers. However, though the Michelson-Morley Experiment had shed doubt on the aether theory, many scientists still believed in it. Furthermore, scientists were still using Newtonian Mechanics to explain the propagation of light. However, Einstein's theory of Special Relativity proposed the independence of the speed of light. This explained the problems of light. The Kennedy-Thorndike Experiment wanted to confirm that the aether did not exist while also showing the consistency of the speed of light. Kennedy and Thorndike built an apparatus that was very similar to that of the Michelson-Morley Experiment. There were two perpendicular beams with one key difference: The beams had uneven lengths. So when the beams recombine after traveling on their distances, the interference pattern of the light waves should be more clear. When the peaks of the waves align with each other, a constructive interference is formed causing bright fringes. When the peaks align with the troughs, there is a destructive interference causing dark fringes. Partial interference causes the fringes to have different intensities. At any point in time, there should be an interference pattern. Since the experiment was also repeated throughout the year, this led to different velocities of the observer because the Earth changes speed as it moves around the Sun. If they observed that the time difference, or in this case a different interference pattern, between the light beams was the same throughout the year, then that would disprove the consistency of the speed of light (7).

To put this into simpler terms, we can use the plane analogy again. Let point A be the position the plane or rather light beam starts from. A beam goes down path AB with length L. Another beam goes down path AC with length $L + \Delta l$. The round trip time for the light beam that goes down AB should be $2L/c = t$. Note that in this case, c represents the speed of light in the laboratory reference frame and not a preferred reference frame. The round trip time for the light beam that goes down AC should be $(2L + 2\Delta l)/c$. Subtracting AB from AC gives $(2\Delta l)/c$. Since there is a time difference, there must also be an interference pattern. In the KT Experiment, the light used had a period of $T = 1.820 \times 10^{-15}$seconds. We can use the formula $\Delta t/T$ to find the number of periods that passes which is equal to the fringe shift. We can use the expression from the previous part to find Δt. In the experiment, $\Delta l = 16\ cm$. In calculating Δt, we have, $(2 \times 16)/(3 \times 10^{8}) = 1.06 \times 10^{-9}$ seconds. Dividing this by T, yields 582417.58 periods.

Since the experiment was performed multiple times throughout the year, if the difference in periods did not change, that number of periods was a result of the unequal path lengths and not the changing velocity of the observer. We can create a formula that puts together all the previous expressions to show the variations of the speed of light c due to changes in the observer's velocity. In the formula $n = \Delta t/T$, it can be rewritten as $n = (2\Delta l)/c/T$ which gives $(2\Delta l)/(nT) = c$. Since Δl and T are constants, if n changes, it would mean the speed of light is dependent on the observer's velocity.

Kennedy and Thorndike came to the conclusion that the speed of light is independent of the observer's velocity. However, compared to modern experiments, the margin of error was relatively large. That's where a modern recreation of the KT experiment by C. Braxmaier, H. Müller, O. Pradl, J. Mlynek, and A. Peters comes in. Their experiment tries to recreate the KT experiment with greater accuracy in measuring A. The KT experiment had some errors in calculating the period over the year. Interferometry (which was used in the original MM and KT experiment) relies on our ability to count the fringes of a shift to measure the speed of light which limits accuracy. Optical resonators (which are used in modern recreations of the MM and KT experiment) on the other hand measure the speed of light by measuring the frequency of a laser which allows for greater precision.

V. Tests of Relativity Using a Cryogenic Optical Resonator

The paper starts off by saying that if special relativity is incorrect, there is a velocity dependence of the speed of light, and therefore the aether theory is correct, then it follows that: $(c(v, \theta))/c = 1 + A(v^{2}/c^{2}) + B(v^{2}/c^{2})sin^{2}\theta$ (4). The Michelson-Morley Experiment had already shown that the direction light moves in has no effect on its speed. A modern recreation of the experiment which also used an optical resonator determined that $|B| < 5 \times 10^{-9}$ which is close enough to 0. However, in order to disprove the aether theory fully and prove the velocity independence of the speed of light, it is necessary to show that $A = 0$. While the Kennedy-Thorndike experiment showed that $|A| < 5 \times 10^{-2}$ (4), in order to increase our understanding of the validity of special relativity, it is necessary to improve this accuracy. Their paper omits many of the details, so this paper will explain and check some of the calculations.

The apparatus used in the paper is called the Cryogenic Optical Resonator (CORE). It measures the speed of light by measuring the changing frequency of the lasers. The changing frequency of the lasers is called V_{res}, and is analogous to $c(v)$. We can get rid of θ in $c(v, \theta)$ because the Michelson-Morley Experiment already proved that the direction in which light travels does not affect its speed. The paper also uses the symbol V_{mol} which is a reference laser, which is analogous to c. Simplifying the first equation given the new information gives: $(c(v) - c)/c = A(v^{2}/c^{2})$. Since c is the reference speed of light, we can replace the left side of the equation to fit the variables of the experiment. $(V_{res} - V_{mol})/V_{mol} = A(v^2/c^2)$. Solving for A gives $|A| = |c^2/v^2| \times (V_{res} - V_{mol})/V_{mol}$. We can now plug in the values of the variables. To

find the upper and lower bounds of v, we can look at equation (2) from their paper. When the value of $sin(\theta) = -1$, the magnitude of $v = 3.4 \times 10^5$. And when $sin(\theta) = 1$, the magnitude of $v = 4.0 \times 10^5$. According to page 3 of the paper, the maximum magnitude of $V_{mol} = 2.82 \times 10^{14}$Hz. $V_{res} - V_{mol}$ can also be found by looking at the highest point in Figure 2 of the original paper. It shows that $V_{res} - V_{mol} < 8 \times 10^3$Hz. c is just the reference speed of light or 3×10^8m/s. By plugging in this information and using the lower bound of v, we get that the magnitude of A is at most 2.2×10^{-5}. Table 2 shows both hypothetical and measured values of V_{res} and their corresponding $|A|$.

TABLE 2	
V_{res} - Frequency of laser	Upper Bound of $\|A\|$
$6882.05 + 2.82 \times 10^{14}$	1.9×10^{-5}
$7244.27 + 2.82 \times 10^{14}$	2.0×10^{-5}
$7606.48 + 2.82 \times 10^{14}$	2.1×10^{-5}
$7968.69 + 2.82 \times 10^{14}$	2.2×10^{-5}
$100.00 + 2.82 \times 10^{14}$	2.7×10^{-7}
$10000.00 + 2.82 \times 10^{14}$	2.8×10^{-4}

VI. Conclusion

By unifying these experiments under the Mansouri-Sexl test theory, this paper provides a clear reference for those trying to understand the background of the invariance of the speed of light. This paper provides both historical and modern results while reanalyzing everything in terms of the Mansouri-Sexl test theory. This in turn, reduces the need to extensively search across multiple different sources which results in easier access to information.

Works Cited

1. Newton, Isaac. "Opticks." *Project Gutenberg*, Project Gutenberg, 23 August 2010, https://www.gutenberg.org/cache/epub/33504/pg33504-images.html. Accessed 10 March 2025.
2. Young, Thomas. "Double-slit experiment." *The Royal Society*, The Royal Society, 1 January 1804, https://royalsocietypublishing.org/doi/10.1098/rstl.1804.0001. Accessed 10 March 2025.
3. Resnick, Robert, and David Halliday. *Basic concepts in relativity and early quantum theory*. 2nd ed., Wiley, John Wiley & Sons Inc, 1985. *Open Library*, https://openlibrary.org/books/OL2859485M/Basic_concepts_in_relativity_and_early_quantum_theory. Accessed 1 December 2024.
4. Braxmaier, C., et al. "Tests of Relativity Using a Cryogenic Optical Resonator." *Physical Review Letters*, vol. 88, no. 1, 2001, p. 4. *Wayback Machine*, https://web.archive.org/web/20210323200106/http://www.exphy.uni-duesseldorf.de/Publikationen/2002/Braxmaier-2002-PRL10401.pdf. Accessed 31 August 2024.
5. Michelson, Albert A., and Edward W. Morley. "Michelson–Morley experiment." *American Journal of Science*, vol. S3-34, no. 203, 18187, p. 12. *American Journal of Science*, https://ajsonline.org/article/62505. Accessed 10 March 2025.
6. Taylor, Edwin F., and John A. Wheeler. *Spacetime Physics*. 2nd ed., W. H. Freeman and Company, 1992.