The effect of cooking container dimensions and mass on heat transfer rate in sous vide cooking

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Introduction

This research paper aims to explore the effect of container size and mass on the cooking time in sous vide cooking, focusing on the physical principles of fluids and thermodynamics. Sous vide, which in French means “under vacuum,” is a simple cooking technique in which food is sealed in a bag and placed in a water bath at a specific, constant temperature for hours, or even days, to slow-cook (Baldwin, 2012). This process often involves an immersion circulator, which heats and circulates the liquid in which the vacuum-sealed food is submerged. Convective heat transfer is a key focus, as the fluid flow differs by the size of the container, affecting the convection currents of the liquid in the cooking process.

Convective heat transfer for cooking is well understood mostly for ovens, pressure cooking, etc., which are considered to be conventional cooking methods. However, sous vide, being a relatively new cooking method, is overlooked. With water being more dense than air, using liquid may allow more efficient and precise cooking. Therefore, looking deeper into and researching ways to improve this machine may boost efficiency and further improvements in the cooking world (Julabo, n.d.). Therefore, this investigation aims to examine the effect of the size and mass of a cooking container in relation to the food’s rate of heat transfer.

Even outside of this specific field, convective heat transfer is used in various ways. This may include water cooling, which is used in various systems, such as car engines and air conditioners.

Background Information

Convective heat transfer is categorized into two primary types: forced and natural convection (Convective Heat Transfer, 2003). During a forced convection, the fluid is forced to flow over the surface by external means, such as an engine or a pump. A key component controlling the rate of heat transfer during forced convection is the Reynolds number, which is a dimensionless quantity that measures the fluid’s ratio of inertial forces to viscous forces. However, as the investigation only uses one type of liquid — water — it does not significantly affect the changes in the time constant. Natural convection, on the other hand, occurs when fluid motion is driven solely by buoyancy forces that result from density differences caused by temperature gradients within the fluid. For natural convection, the Grashof number is the key dimensionless parameter, representing the ratio of buoyancy to viscous forces. This investigation focuses on the mixed convection formed from the immersion circulator – mixed convection refers to situations where both natural and forced convection contribute to the heat transfer process.

Another component for consideration is the boundary layer thickness near the specimen. In regions where the flow is slower, the boundary layer becomes thicker, reducing the heat transfer rate to the food. In contrast, faster flow reduces boundary layer thickness, enhancing heat transfer. A thinner boundary layer is particularly important in sous-vide cooking to ensure that the heat penetrates the food efficiently and cooks uniformly. (Maruyama and Ishizaki, 1988).

In order to actually investigate the time it takes for the meat load to heat up to the equilibrium temperature, Newton's law of cooling (heating)—the rate at which a body loses heat is proportional to the difference in temperature between the body and the surroundings—is used to model heat transfer. This empirical model stands true and represents data most accurately during substantial temperature differences in forced convection, which is hugely attributed to the sous vide cooking method (Newton's Law of Cooling). Regarding the definition, Newton's law of cooling (heating) can be defined by the following formula:

$$\frac{dQ(t)}{dt} = -k(T - T_{ambient}) \quad \text{(Eq.1)}$$

where $\frac{dQ(t)}{dt}$ is the rate of heat transfer per unit of time (SI unit: W), k is the positive constant dependent on the area and the heat transfer coefficient of the body, T is the temperature of the body (SI unit: K), and $T_{ambient}$ is the temperature of the environment (SI unit: K); $T_{ambient}$ is assumed to be a constant value since the ambient environment is treated as an infinite thermal reservoir (Silva, 2021). However, for the purposes of the investigation, it is necessary to readjust the formula to identify the rate of heat transfer of the load while reaching the equilibrium temperature. Therefore, the time constant (τ) is crucial in determining the rate at which the specimen's (food) internal temperature increases to reach the equilibrium temperature with the heated medium (water). Therefore, including the time constant variable, the formula is given by:

$$\frac{dQ(t)}{dt} = -\frac{1}{\tau}(T - T_{ambient}) \quad \text{(Eq. 2)}$$

$$T = T_{ambient} + (T_0 - T_{ambient})e^{-\frac{t}{\tau}} \quad \text{(Eq. 3)}$$

where τ is the time constant (SI unit: s) and T_0 is the initial temperature (SI unit: K) (Mandell, 2013). τ depends on the food's mass, surface area, specific heat capacity, and the heat transfer coefficient of the cooking container. Therefore, a key component of determining τ is the heat transfer coefficient of the container. In theory and practice, measurements regarding heat transfer are often associated with the discovery of a constant value of the "heat transfer coefficient". The heat transfer coefficient is empirically defined from Newton's law of cooling, shown through its relationship with the time constant in regards to the lumped capacitance method (Silva, 2021); the model assumes the internal temperature of the solid to be spatially uniform (VPS, 2011):

$$T = T_{ambient} + (T_0 - T_{ambient})e^{-\frac{t}{\tau}} \quad \text{(Eq. 4)}$$

$$T = T_{ambient} + (T_0 - T_{ambient})e^{-\frac{Ak}{mc_p}\cdot t} \qquad \text{(Eq. 5)}$$

$$\tau = R_t C_t = \rho c_p V \cdot \frac{1}{Ak} = \frac{mc_p}{Ak} \qquad \text{(Eq. 6)}$$

where R_t is the resistance to convection heat transfer (SI unit: K/W), C_t is the thermal capacitance of the solid (SI unit: J/K), ρ is the density of the solid body (SI unit: kg/m^3), V is the volume of the solid body (SI unit: m^3), m is the mass of the solid body (SI unit: kg), c_p is the specific heat capacity of the load (SI unit: $J/kg \cdot K$), A is the surface area of the container (SI unit: m^2), and k is the heat transfer coefficient of the container (SI unit: $W/m^2 \cdot K$) (Bahrami, 2009); the thermal capacitance C_t refers to the ability of a material to store heat. It is assumed to be a constant value since the meat's internal temperature is assumed to be uniform in this model (Birur et al., 2003).

The heat transfer coefficient (k) is a key parameter in this paper – the coefficient quantifies the rate at which heat is transferred between a solid surface and a fluid per unit area per unit temperature difference. It is largely influenced by the fluid properties, fluid velocity, surface roughness, and shape, and the types of convection. In the context of sous-vide cooking, the varied container sizes and mass may affect the water flow velocity, potentially influencing changes in the thickness of the thermal boundary layers around the food and therefore changing the heat transfer coefficient (Yıldız and Liu, 2018).

The investigation is conducted by varying the size and mass of containers and observing the change in the internal temperature of the meat being cooked for each container until it reaches an equilibrium temperature with the surrounding liquid; this is done with a temperature probe. In order to minimize systematic errors, ground beef balls with equal shape, surface area, and density (mass), along with consistent fat content, are used for the meat in the plastic bag.

Methodology

To conduct the experiment, the room temperature was set to a standard range of 20–22 °C. Then, using an electronic scale, 50 g of ground pork was measured and shaped into a sphere while wearing latex gloves to avoid contamination. A total of twenty-five 50g meatballs have been made to perform five trials for five containers. Regarding Figure 1, a marker pen was used to mark the depth of the temperature probe to where it should stick in the meatball to ensure consistent positioning across trials. The pork meatball was placed in the center of a sealed Ziploc bag to prevent intersection with the fluid and solely see the effect of the sous vide cooking method. Each container was filled to 80% capacity with cold water, and an immersion circulator was attached. A vertical stand with a horizontal rod was positioned above the container to keep the specimen at the center of the container in order to equally observe the effect of convection across trials. A hook-type stainless steel clamp on the vertical stand held the Ziploc bag so the pork ball was fully submerged but not in contact with the bottom of the container to see the full effect of convective heat transfer. A specimen that contacts the side of the container or is not fully submerged may only see the partial effect of a convective heat transfer. Then, plastic wrap was used to cover the container and temperature logger to reduce evaporation.

The circulator was set to 24.0 °C, and the internal temperature of the pork was monitored until equilibrium was reached. The temperature of the water was set to 24.0 °C in order to control the initial internal temperature of the load. Once the meat reached 24.0 °C, as indicated by the data logger, the temperature of the immersion circulator was increased to 69.0 °C and the data collection began. The recording stopped when the internal temperature stabilized at 69.0 °C. This process was repeated for each container five times, respectively for the ones with the mass-to-cross-sectional area ratio ($\frac{m}{A}$) of 64.17 kg/m², 65.76 kg/m², 74.41 kg/m², 78.24 kg/m², and 78.33 kg/m².

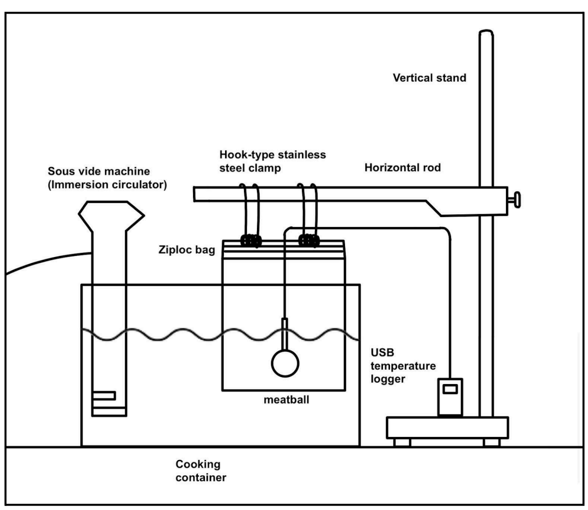

Figure 1: Diagram of the experiment, where the USB temperature logger records the internal temperature of the meatball submerged in the water and the mass and size of the cooking container change as a variable.

Results

This section shows the results of the experiments conducted to show the impact of the sous-vide container's mass-to-size ratio on the cooking time. Specifically, it investigates the time constant for each container and validates the data by looking into the relationship between the specific heat capacity and the heat transfer coefficient, which composes the time constant, regarding the lump capacitance method.

Figure 2 shows the quantitative raw data collected from the USB temperature logger over a 36-minute period - until the meat reaches the equilibrium temperature of 69.0 °C. It displays the data collected in the first trial of the first container, which has a ratio value of 64.17 kg/m², as a representation of the entire trial. The following process is repeated five times for each container, making up a total of 25 trials. Consequently, the time in which the meat is cooked is calculated by fitting Newton's law of cooling to the data and allowing the time constant for each trial to be determined:

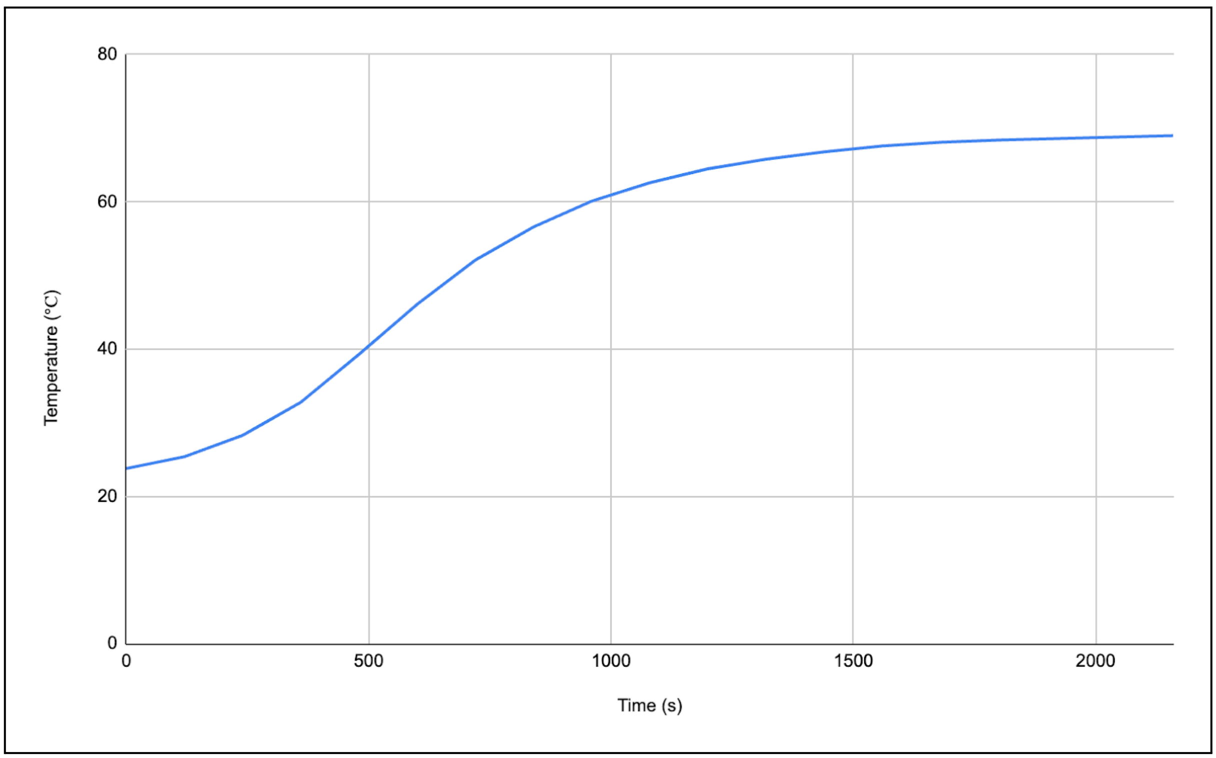

Figure 2: Graph of the temperature (°C) vs. time collected by a temperature data logger for container 1 trial 1

The τ is found for each container and trial after applying Newton's Law of Cooling formula in Python. The relationship between τ and the $\frac{m}{A}$ of each container is shown in Figure 3.

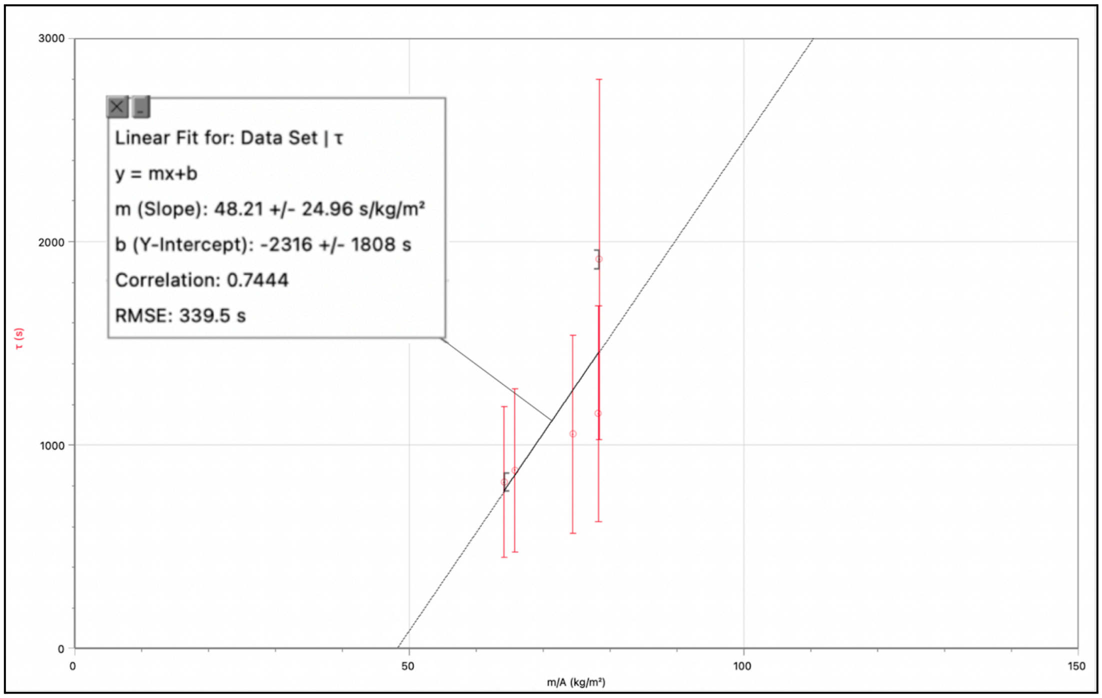

Figure 3: Graph of the mass/area (kg/m^2) vs. average time constant (s) for the five containers

In order to validate the relationship between τ and the ratio of $\frac{m}{A}$, the resulting value is compared to the gradient value from Figure 3, being the specific heat capacity of the load over the heat transfer coefficient of the container, as shown in equations 7 and 8 with its directly proportional relationship:

$$\tau = \frac{mc_p}{Ak} \quad \text{(Eq. 7)}$$

$$\tau/(\frac{m}{A}) = \frac{c_p}{k} \quad \text{(Eq. 8)}$$

According to Figure 3, the value of the specific heat capacity of the load over the heat transfer coefficient of the container ($\frac{c_p}{k}$) is approximately 48.21 with an uncertainty of 24.96 $s/kg/m^2$. The data from the 5th container is considered an anomaly, with its value being greater than $Q3 + 1.5 \cdot IQR$. The linearized gradient, which represents the value of $\frac{c_p}{k}$, therefore has a moderately strong, positive correlation coefficient of 0.744. This assumes that the specific heat capacity of the load is that of water since the mass of pork is significantly smaller than that of water, and the specific heat capacity of the water and the pork does not vary much.

Thus, to find the heat transfer coefficient of the container, the specific heat capacity of water should be divided by the gradient of the linearized graph:

$$k = c_p \cdot \frac{(\frac{m}{A})}{\tau} = \frac{4200}{48.21} = 99.56\, W/m^2K \quad \text{(Eq. 9)}$$

The heat transfer coefficient of a typical forced convection during moderate-speed flow of air over a surface is 100 W/m^2K (Convective Heat Transfer Table, n.d.). With the uncertainties of 24.96 $s/kg/m^2$, the data fits within the range of errors, almost accurately representing the conventional value of the appropriate heat transfer coefficient. Therefore, the correlation between the size of the cooking container and the rate at which the temperature of the specimen rises is shown to be true.

However, this investigation includes several errors; this may be shown as the correlation coefficient of 0.744 does not show a perfect fit of data whilst the heat transfer coefficient does, indicating potential deviations within the study. The inconsistent placement of the temperature probe within the meatball could introduce inconsistent data collection. If the probe was positioned closer to the surface, it would record a faster rise in temperature, leading to a shorter observed cooking time. Conversely, if placed too deep, it might delay the time at which the internal temperature approaches equilibrium, artificially inflating the time constant. Variations in probe positioning would result in inconsistent time constant values, particularly across trials and containers, which contributes to the large range observed in the final time constants. Furthermore, the immersion circulator may not have provided perfectly uniform water flow in every trial. Variations in flow rates, especially at lower Reynolds numbers, could result in inconsistent heat transfer between trials. Additionally, turbulent flow enhances heat transfer, while laminar flow in regions of weaker circulation reduces it, affecting the convection process. This means a non-uniform flow in the water bath would lead to unequal heating across trials, impacting the heat transfer rate and thus contributing to the variation in time constants and the $\frac{c_p}{k}$ value. Potential errors also exist within the methodology. In the methodology, the pork meatballs were initially at a lower temperature due to refrigeration, and the water bath was heated to 24.0°C before increasing to the final target temperature of 69.0°C. Small differences in the pork's initial temperature could affect how quickly it reaches equilibrium with the water. Variability in the meat's starting temperature could affect the calculated time constant, adding to the discrepancies observed between trials. These errors are shown as the linearized graph does not pass through the origin (0,0).

Conclusion

This research demonstrates the significant impact of container size and heat transfer efficiency on the sous-vide cooking process. The observed C/k values, derived from experiments using containers of varying cross-sectional areas, provide insights into how heat transfer occurs in this controlled cooking environment. However, the large range in C/k values, from 1.25 to 350, is indicative of systematic errors that likely influenced the results. Inconsistent probe placement, potential heat loss, variations in water circulation, and measurement inaccuracies contributed to this variability.

Further investigation could be done by considering fluid dynamics. A key factor of sous vide cooking is mixed convection—therefore, it is necessary to discuss the extent to which this immersion circulator is forced convection and the various effects this has on sous vide cooking.

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