Research paper
July 2022
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Research question: How do changing factors like driving frequency, core diameter, and core length affect the inductive reactance of an inductor?

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<u>How do changing factors like driving frequency, coil diameter, and coil length affect the inductive reactance of an inductor?</u>

Introduction:

In the Information Age, we can not live without wireless devices. Wireless devices such as WiFi, telephones, radio, and Bluetooth improve the way humans interact with technology. When we make phone calls, our phones emit specific electromagnetic waves to a receiver or base station. To receive the desired frequency, a filter is required to eliminate unwanted frequencies. An inductor-capacitor (LC) filter and inductor-resistor (LR) filter are two very common filters which use an inductor (and capacitor or resistor) to filter electromagnetic noise. When an alternating current is applied to an inductor the back electromotive force (EMF) produced by the inductor will change accordingly. The back EMF opposes the supplied voltage and the flow of current is limited. According to theory, the inductor's opposition to the flow of current increases with the driving frequency. In this way, the circuit can remove the unwanted frequency components. Here we investigate the effect of different driving frequencies of alternating current, coil diameter and coil length on the inductive reactance of an inductor. We discovered that changing factors like frequency, coil diameter, and coil length can lead to the changes in the inductance of an inductor as well as the induced voltage produced by the inductor. As the inductance of the inductor changes, the inductive reactance varies as well as the effect of the inductor on preventing certain frequencies of alternating current. In the process of the investigation, we will proceed both experimentally and theoretically, comparing the results. Our results demonstrate under what circumstances the inductive reactance has the largest value and this value can play a key role in preventing the effect of electromagnetic noise on wireless systems.

Background information:

1. Electromotive force (EMF)

Electromotive force is the energy per unit charge that is imparted by a non-electrical source. Such a source can be a battery or a generator which can convert other form of energy, such as chemical energy and mechanical energy into electric energy.

The electromotive force is equal to terminal potential difference when there is no current flow. The unit of EMF is volt (V) which is the same as the potential difference.

From Ohm's law, EMF can be written in terms of the internal resistance of battery (r) where:

$$\mathbf{EMF(\varepsilon)=I(r+R)=V+IR}$$

I = current flow in the circuit
R = internal resistance of the battery
R = total resistance in the circuit
V = the terminal potential difference

2. Alternating current (AC)

"Alternating current is an electric current which periodically reverses direction and changes its magnitude continuously with time." (Wikimedia Foundation, 2022, September 5) It first starts from zero, grows to a maximum, decreases to zero, reaches a maximum in the opposite direction, returns to zero, and repeats this cycle. A typical waveform of alternating current is a sine wave.

Compared to direct current (DC) which cannot change the direction, alternating current can transfer energy from a large distance without much energy loss due to resistance. Due to less energy loss, alternating current is usually used for power transmission.

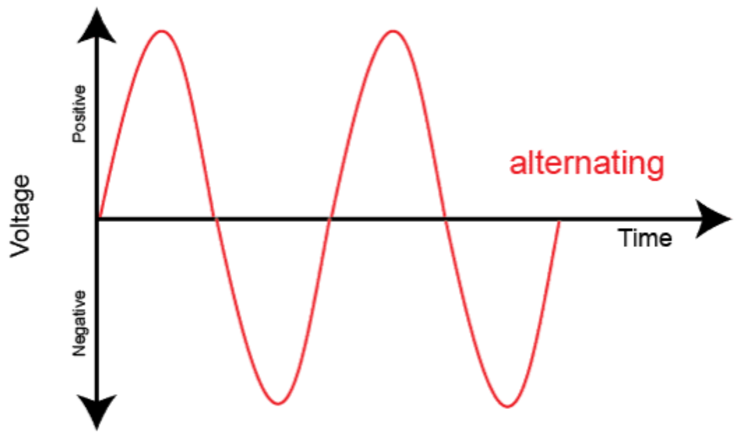

Figure 1. The waveform of alternating current (AC). (Gordon Powers, Retrived 2022)

3. Faraday's law and Lenz's law

Faraday's law states that whenever there is any change in the magnetic field of a coil of wire, a voltage will be induced in the coil. "The change in magnetic field can be produced by moving a magnet towards or away from the coil." (Faraday's law, Retrieved 2022)

The induced voltage can be expressed as:

$$\textbf{Induced voltage}(\varepsilon) = -N \cdot \Delta(BA)/\Delta t$$

N = total number of turns of the loop
Δ(BA) = rate of change of magnetic flux
B = magnetic field strength
A = area of the coil
t = time

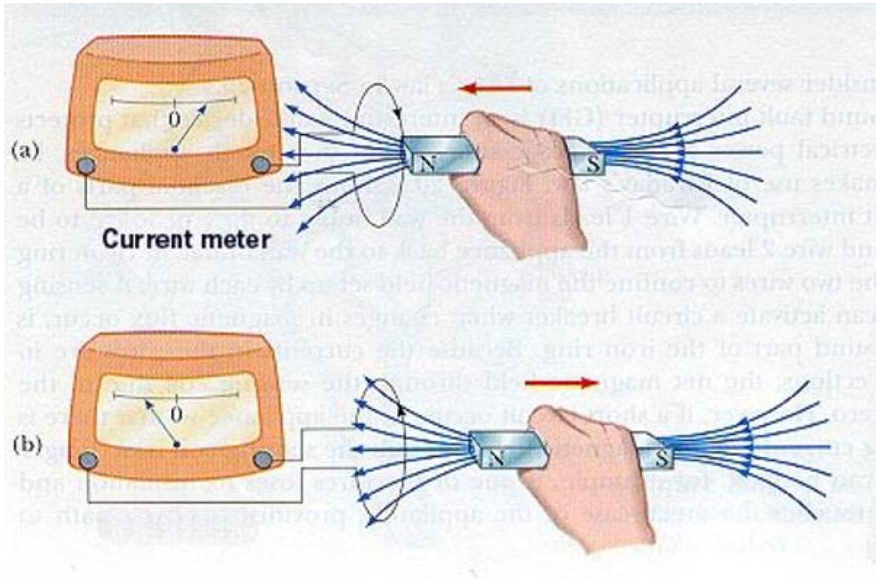

Figure 2. As the magnet is moved towards or away from the coil of wires, a voltage will be induced in the coil. (Daly. B. 2022)

Lenz's law is contained in Faraday's law. It states that the current induced in a circuit due to a change in a magnetic field is directed to oppose the change in flux to keep the magnetic flux constant. As figure 2 shows, when the external magnetic field increases, a current will be induced in the coil, generating an induced magnetic field opposed to the increase. If the external magnetic field decreases, a current and a field will be induced to oppose the decrease. Mathematically, Lenz's law is captured by the negative sign in Faraday's law.

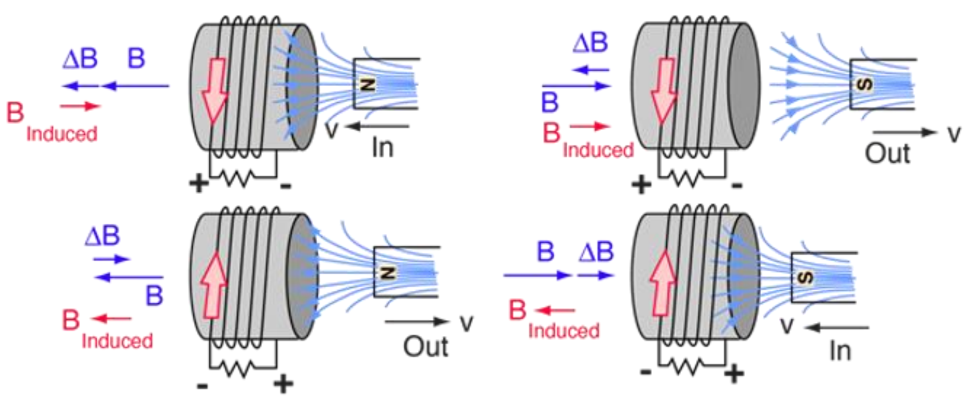

Figure 3. The example of Lenz's law. (Faraday's law, Retrieved 2022)

4. Inductance

Inductance is the property of a circuit element that causes an induced EMF to be generated by a change in the current through the device in order to resist any current change. According to Faraday's law, a change in current causes a change in magnetic field through the circuit. The change in magnetic field through the circuit induces an EMF. This induced EMF will oppose the change in current.

The standard unit for inductance is Henry (H) and the inductance L can be expressed in terms of EMF as:

$$\mathbf{EMF = -L \bullet (\Delta I/\Delta t)}$$

L = inductance
ΔI = rate of change in current

The inductance L can be expressed as:

$$\mathbf{L = (\mu N^2 A)/l}$$

N = number of turns in the coil
μ = permeability of the core material
A = area of coil
l = average length of coil

5. Inductor and filter

An "inductor is an electrical component consisting of coils of wires to store electrical energy in the form of magnetic energy." (Wikimedia Foundation, 2022) When the current flow through it changes, the varying magnetic field induces an EMF. This EMF causes an induced current which has a direction which opposes the change in current. An inductor is usually used to block high frequency alternating currents, as well as allowing direct current and low frequency alternating currents to pass through.

A filter is an electrical circuit which allows the passing of signals of predetermined frequencies and rejects all other frequencies. A filter is normally composed of 2 or more complementary circuit elements; standard filters are LC (inductor and capacitor), LR (inductor and resistor), or RLC (resistor, inductor and capacitor) topologies.

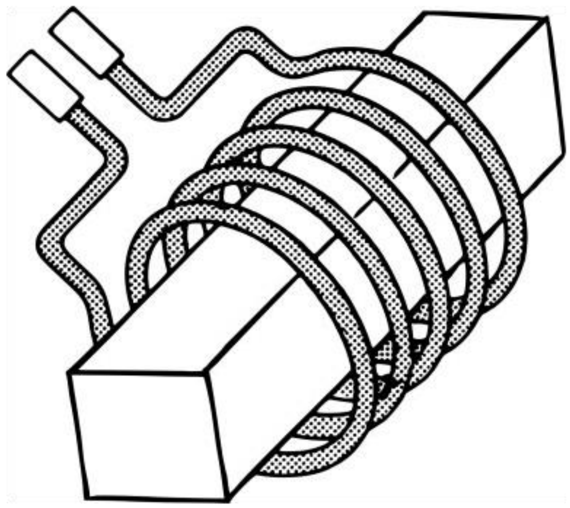

Figure 4. An inductor (Take Online Courses. Earn College Credit. Research Schools, Degrees & Careers, Retrieved 2022)

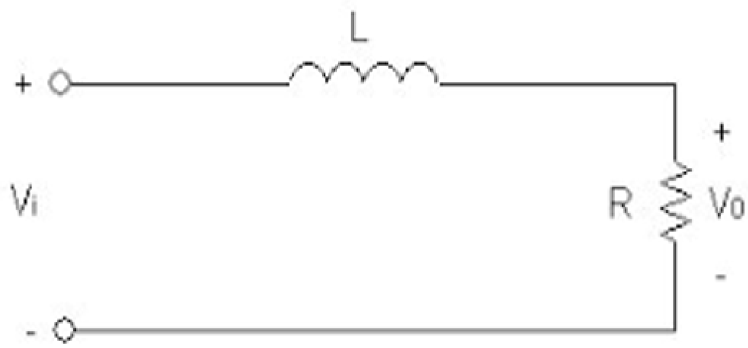

Figure 5. A filter of LR circuit (Online courses,Retrieved 2022)

6. Inductive reactance

Inductive reactance (X_L) is the opposition to the flow of alternating current by an inductor. Compared to resistance which can oppose the flow of direct current, reactance can block alternating current. Just like resistance, inductive reactance is expressed in Ohms (Ω).

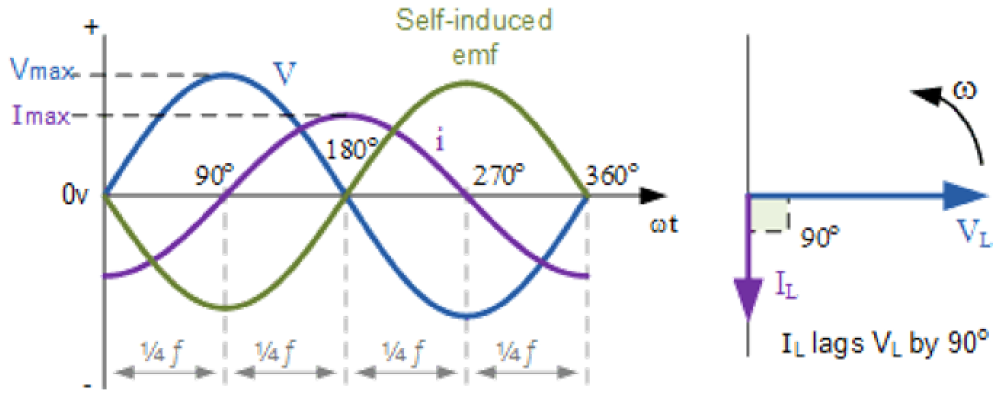

Figure 6. The relationship between the voltage across the inductors and the current flows. (Basic electronics tutorials and Revision, Retrieved 2022)

“In the purely inductive circuit, the current flows through the inductors lags the voltage across it by 90° (π/2).” (Basic electronics tutorials and Revision, Retrieved 2022) In other words, the voltage leads the current by 90°. Therefore, when the voltage is 0°, the current will be -90°. If the voltage waveform of the alternating current is classified as a sine wave, then the current waveform can be classified as a negative cosine wave. Thus, the value of current can be expressed as being:

$$\mathbf{I_L = I_{max} \bullet \sin(\omega t - 90^\circ)}$$

ω is in radians per second and t is in seconds.

Therefore, if we know the value of VL, then the IL must lag by 90°. Likewise, if we know the value of IL, then the value of VL must lead by 90°. The value of inductive reactance XL can be expressed in terms of VL and IL as:

$$\mathbf{X_L = V_L / IL = \omega L}$$

L=Inductance

However, in reality, it is impossible to have a purely inductive coil which has a pure inductance. All coils and solenoids have a certain amount of series resistance, as shown in the figure below. Since the current and voltage are in phase for a resistor and 90° out of phase for an inductor, vector addition must be used determine the voltage across each component (VR and VL) at a given time. An example of vector addition is shown below for an LR circuit.

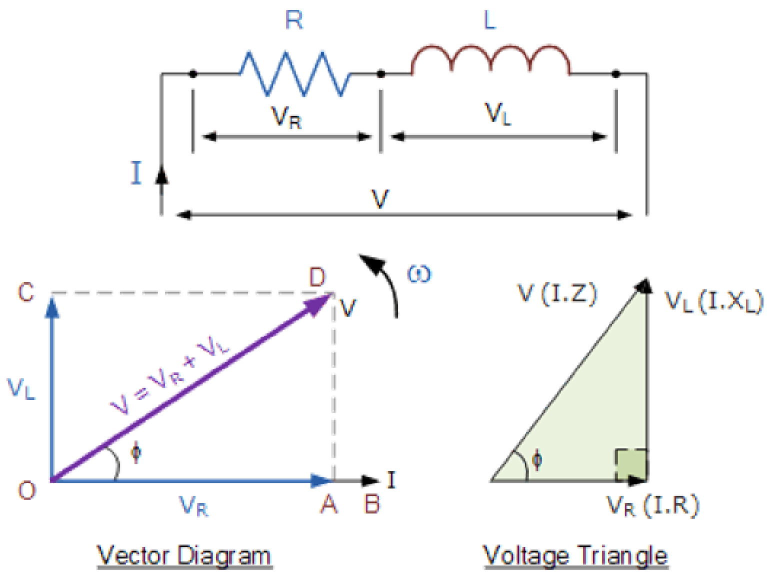

Figure 7. The vector addition in LR circuit. (Basic electronics tutorials and Revision, Retrieved 2022)

From the vector diagram above, line OB represents the current flows through the circuit, line OA represents the voltage across the resistor (VR) which is in phase with the current and line OC represents the voltage across the inductor (VL) which is 90° in front of the VR. Line OD gives the resultant supply voltage of the circuit. The voltage triangle is derived for Pythagoras theorem and is given as:

$$V^2 = V_L^2 + V_R^2$$

Experimental Setup

To investigate how changing driving frequency, coil diameter and coil length affect the inductive reactance of an inductor, An LR circuit was made and the voltage across the inductor and current through the inductor was measured. The circuit was driven by a function generator with an internal source impedance of 50 ohms. Three inductors of the same diameter but different lengths and three inductors of the same length but different diameters were tested.

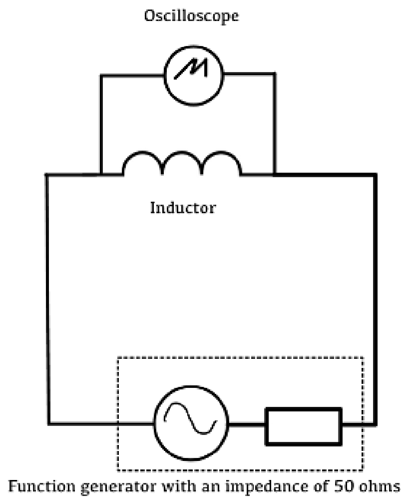

Figure 8. The circuit used in the experiment

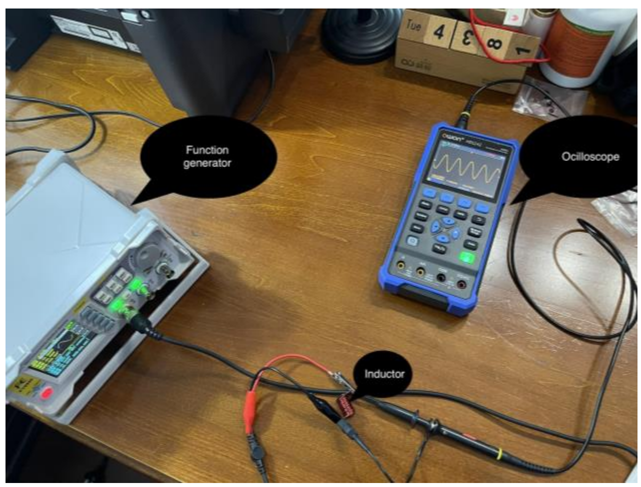

Figure 9. The apparatus of the experiment

Figure 10. The different inductors used in the experiment

Equipments	Version	Operating frequency
DDS function waveform generator	FY6900	0 - 20MHz
Handheld oscilloscope	HDS242	0 - 40MHz

Above is the name and the version of the equipment used in the experiment. The function generator can create sinusoidal signals with frequencies ranging from 0 to 20 MHz, and the oscilloscope can measure frequencies from 0 to 40 MHz.

To measure the effect of driving frequency on inductive reactance, an inductor with ferrite rod with a diameter of 10mm and a length of 20mm was used. The peak-to-peak voltage (Vpp) of the function generator was set to 5V and seven different frequencies were tested: 2 MHz, 2.5 MHz, 3 MHz, 3.5 MHz, 4 MHz, 4.5 MHz, and 5 MHz.

Constant	Variable
Length (10mm)	Driving Frequency (2 MHz to 5 MHz)
Diameter (20mm)	
V pp (5V)	

To measure the effect of the coil length on inductive reactance, the frequency 4 MHz drive frequency and Vpp = 5V of the function generator was kept constant. Three different length inductors were tested: 20 mm, 25 mm, and 30 mm.

Constant	Variable
Driving Frequency(2MHz)	Length (20 mm, 25 mm, 30 mm)
Diameter (10mm)	
V pp (5V)	

To measure the effect of the coil diameter, the driving frequency, length of the ferrite, and the driving voltage were kept constant. Four different diameters of ferrite cores were tested: 4mm, 5mm, 6mm.

Constant	Variable
Driving Frequency(2MHz)	Diameter (4 mm, 5 mm, 6 mm)
Length(25mm)	
V pp (5V)	

To obtain the value of inductive reactance, the peak-to-peak voltage across the inductor was measured, and vector addition was used to find the voltage across the 50 Ohms impedance of the generator. Then, the ratio of the voltage was used to calculate the reactance of the inductor. The formula can be expressed as:

$$\mathbf{V_R = \sqrt{(V^2 - V_L^2)}}$$

$$\mathbf{X_L = R \bullet V_L/V_R}$$

V = Total voltage supplied
V_R = Voltage across impedance in the function generator
V_L = Voltage across inductor
X_L = Inductive reactance
R = Impedance of generator (50 Ohms)

Experimental Data

1. Factor 1: Driving frequency of the alternating current

Constant: length (20 mm), diameter (10 mm), function generator voltage (5 Vpp)

Frequencies	Voltage (peak-to-peak) across the inductor	Voltage across the 50 Ohms	Inductive reactance
2.0 MHz	3.68 V	3.38 V	54.4 Ω
2.5 MHz	4.04 V	2.95 V	68.7 Ω
3.0 MHz	4.24 V	2.65 V	80.0 Ω
3.5 MHz	4.40 V	2.37 V	92.8 Ω
4.0 MHz	4.50 V	2.18 V	103.2 Ω
4.5 MHz	4.60 V	1.96 V	117.3 Ω
5.0 MHz	4.68 V	1.76 V	132.9 Ω

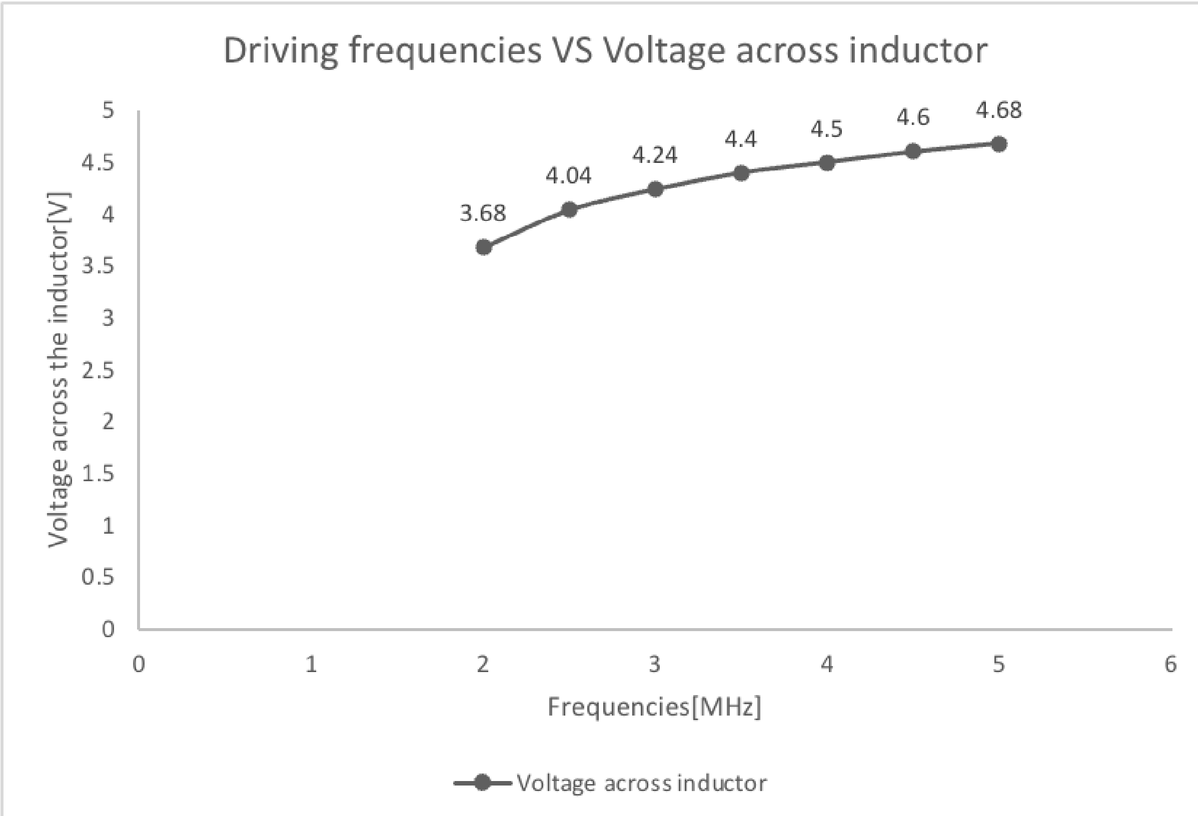

Figure 11. The relationship between Driving frequencies and voltage of the inductor

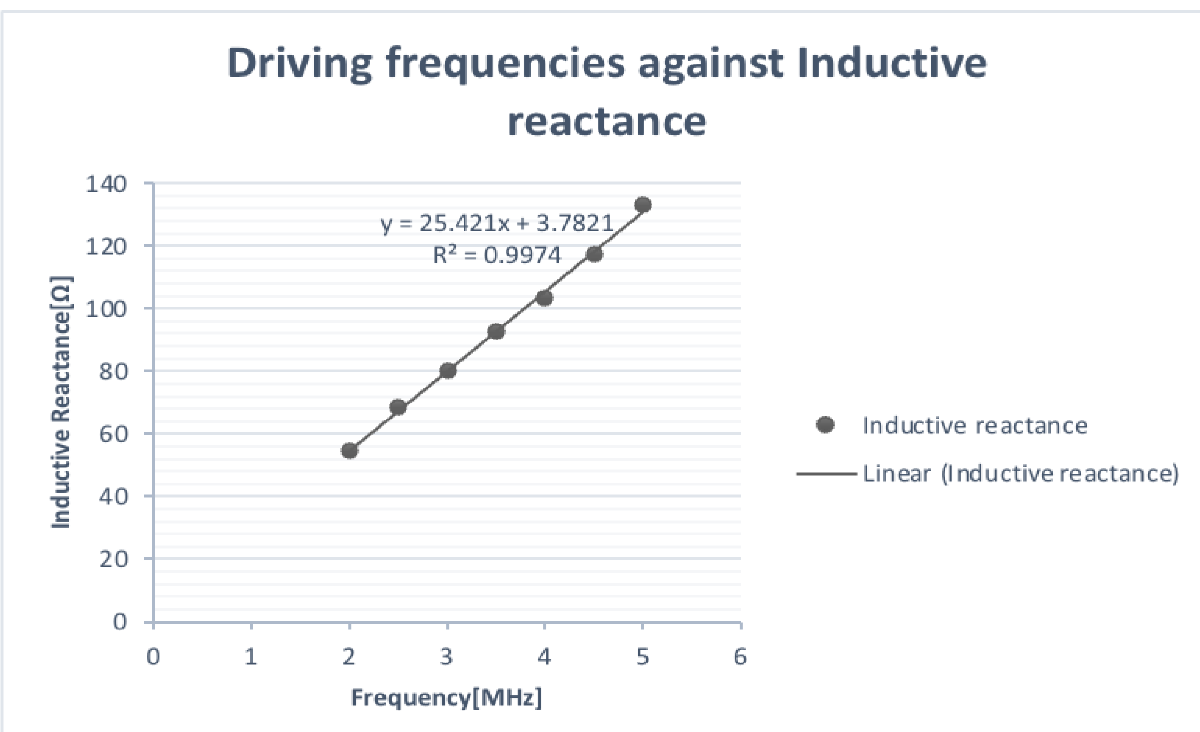

Figure 12. The relationship between driving frequencies and inductive reactance

As shown in the graph above, when the driving frequency of the alternating current increases by 0.5 Hz, the inductive reactance of the inductor increases by approximately 13 Ohms. Since the relationship between inductive reactance and frequency is a straight line, driving frequency is proportional to inductive reactance. Higher frequency leads to higher inductive reactance.

According to graph above, it is shown that the slope of the equation is 25.42 Ω/MHz. Using the equation of inductive reactance which is X=2π*L*f, it follows that the slope 25.42 Ω/MHz, represents 2π*L and L is measured as 4.33 H with a LCR meter. Therefore, plugging in the value 4.33 H into 2π*L, the value of inductance reactance of 27 Ω/MHz is obtained, which is close to the value of 25.42 Ω/MHz obtained from the experiment.

2. Factor 2: The length of the coil

Constant: diameter (10 mm), driving frequency (4 MHz), function generator voltage (5 Vpp), Number of turns of wires (14 turns)

Length	Voltage (peak-to-peak) across the inductor	Voltage across the inductor	Inductive reactance
30mm	4.48V	2.22 V	100.9 Ω
25mm	4.56 V	2.05 V	111.2 Ω
20mm	4.64 V	1.86 V	124.7 Ω

Figure 13. The relationship between length of core and voltage across the inductor

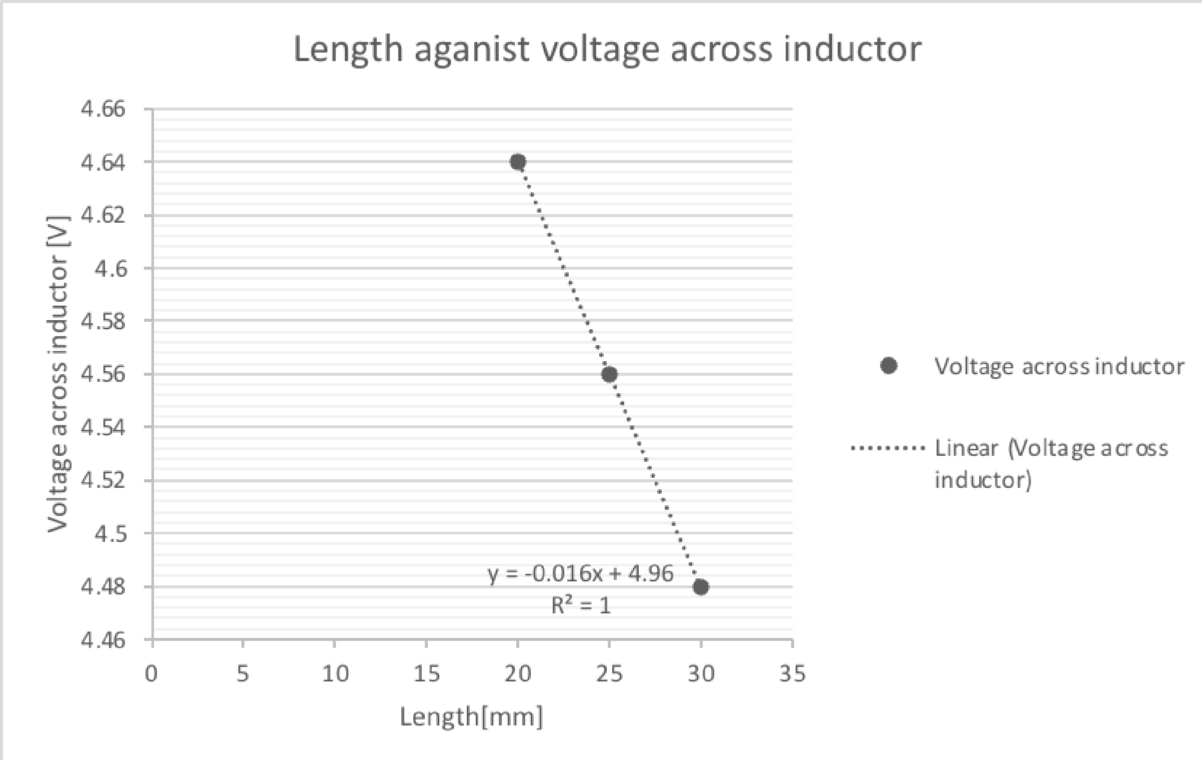

Figure 14. The relationship between length of the core and inductive reactance

As shown in the graph above, the inductive reactance is inversely proportional to the length of the coil. Larger length of the coil means smaller average length between turns, which will lead to smaller inductive reactance.

3. Factor 3: The diameter of the coil

Constants: the length of the ferrite core (25 mm), the driving frequency (4 MHz), function generator voltage (5 Vpp), turns of wire (14 turns)

Diameter	Voltage (peak to peak) across the inductor	Voltage across the inductor	Inductive reactance
4 mm	3.96 V	3.05 V	64.9 Ω
5 mm	4.28 V	2.58 V	82.9 Ω
6 mm	4.60 V	1.96 V	117.3 Ω

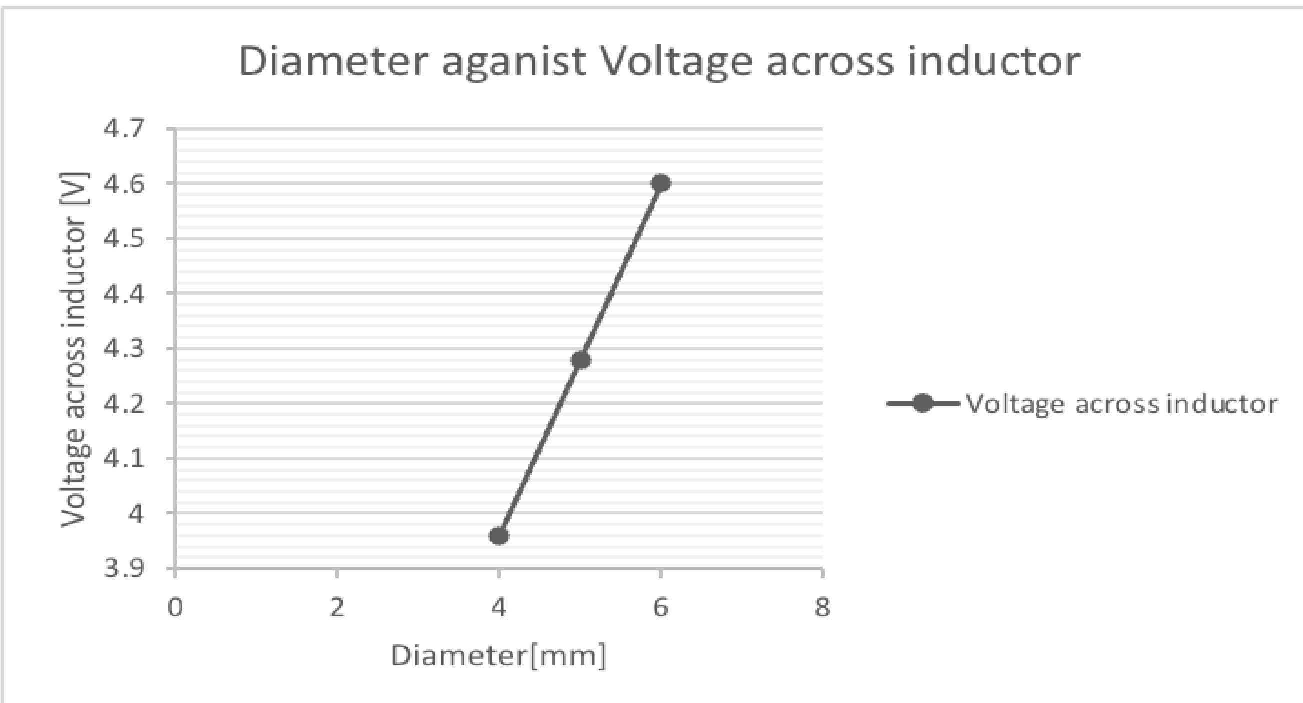

Figure 15. The relationship between diameter of the core and voltage across the inductor

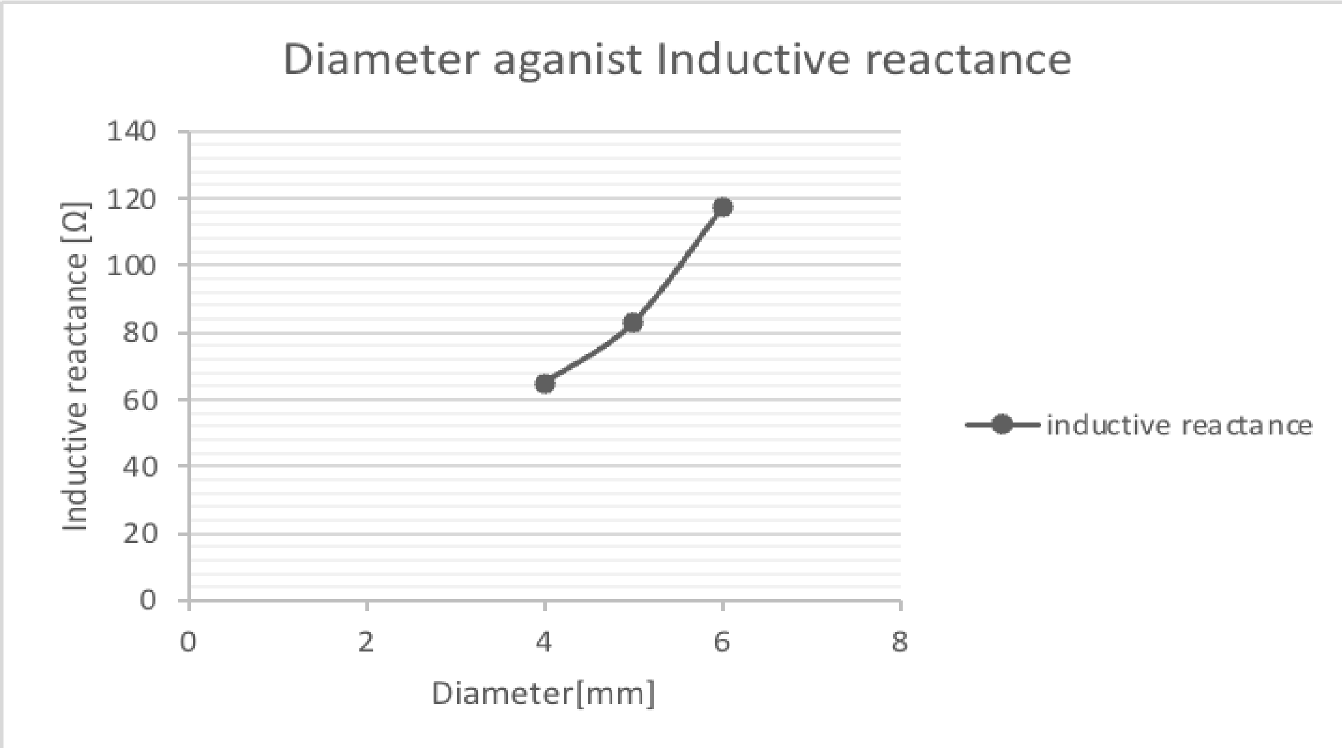

Figure 16. The relationship between diameter of the core and inductive reactance

From the graph above, the relationship between length of coil and inductive reactance is nearly a quadratic function, which means that the inductive reactance is proportional to the square of the diameter of the core. When the length of coil increase, the inductive reactance will also increase significantly.
From the three experiments above, it is obvious that inductive reactance is proportional to the driving frequency, inversely proportional to the core length, and proportional to the square of the core diameter.

Experiment Evaluation

There are still some factors that may influence the data collected which leads to inconsistencies with theory.

1. The impedance of the function generator is not exactly 50 Ohms. Instead, it has a tolerance of 50 Ω +10% to 50 Ω - 10%. Therefore, the ratio of the VL/VR is not constant all the time and the inductive reactance calculated this way is an ideal value.

frequency characteristic

Product Model	FY6900-20M	FY6900-30M	FY6900-50M	FY6900-60M
Frequency range of Sine wave	0~20MHz	0~30MHz	0~50MHz	0~60MHz
Frequency range of square wave	0~15MHz	0~20MHz	0~25MHz	0~25MHz
Frequency range of other waves	0~10MHz	0~10MHz	0~10MHz	0~10MHz
Minimum adjustable width of pulse wave	20ns			
Minimum Frequency Resolution	1μHz			
Waveform Length	8192 points (8K points) *14Bits			
Waveform sampling rate	250MSa/s			
Vertical Resolution of Waveform	14 Bits			
Amplitude range (peak to peak)	frequency≤5MHz	5MHz< frequency≤10MHz	10MHz< frequency≤20MHz	frequency >20MHz
	1mVpp~24Vppp	1mVpp~20Vp	1mVpp~10Vpp	1mVpp~5Vpp
Amplitude resolution	1mV			
Output impedance	50Ω±10% (typical)			
Bias Adjustment Range	frequency≤20MHz:±12V; frequency >20MHz: ±2.5V			
Minimum Bias Resolution	1mV			
Phase adjustment range	0~359.99°			
Minimum phase resolution	0.01°			
display	2.42.4 inches TFT color liquid crystal display			
Interface	115200 bps			
Power Supply	AC100V~240V			
environment condition	temperature: 0~40°C humidity: <80%			

·**Frequently Asked Questions**

Figure 17. The information of function generator

As shown in the table above, the impedance of the source can fluctuate from 45 Ohms to 55 Ohms, which leads to random error of the calculation. Moreover, the voltage supply isn't perfectly stable, which could lead to inaccurate calculation of inductive reactance.

2. The values of peak-to-peak voltage varied on the oscilloscope, thus, the average value was used. In this way, random errors occurred when taking the average value of the inductive reactance.

3. Since the wires around the inductor were wound by a human, it is inevitable that the distance between each turn was variable and the winding density fluctuated. Therefore, when comparing inductors with different diameters or different length, the practical value

of their inductance could be slightly different from the theoretically predicted value. This may explain why the graphs plotted don't perfectly fit the theoretical expectations.

Conclusion

This experiment tested the effect of driving frequency, coil length and coil diameters on inductive reactance of an inductor. The results show that inductors with largest frequency, smallest length and largest diameter have maximum reactance. These results can play an important role in preventing unwanted electromagnetic noise in wireless devices.

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