Microscale Black Hole Simulation: Effects on Quantum Particles

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Abstract

Trends in the behaviors of a particle near a black hole on a quantum scale remains largely unspecified in the field of astrophysics. Their nature and application are a topic of ongoing research and debate. Simulating different variables of a particle as it nears a black hole (i.e. radial position, polar angle, azimuthal momentum) can reveal significant relationships in the data. However, due to the Heisenberg Uncertainty Principle – which states that we cannot know both the position and speed of a particle, such as a photon or electron, with perfect accuracy – the radial position and the momentum of the simulated particle cannot be simultaneously determined in the presence of a genuine black hole at a quantum level. Keeping this limit in mind, the resulting values and trends for both radial, polar, and azimuthal momentums and radial position as a function of time are individually analyzed within the scope of this simulation and used to establish general experimental concepts able to be broadly applied, filling in knowledge gaps in the realm of theoretical physics.

Introduction

Understanding black holes and their effects of gravity on objects has proved to be a significant challenge in astrophysics. Attempts to connect the concepts of quantum mechanics with Einstein's General Relativity to find the "theory of everything" – a theory that unifies these two seemingly incompatible fields (Loop Quantum Gravity Theory or String Theory, for example) – involves complex modifications made to classical theories in order to better understand the behavior of particles at the quantum level. Being able to prove such a theory would help tremendously in understanding the concepts of black holes and singularities. However, a definite theory has yet to be established.

A black hole singularity is an infinitely dense, infinitely small point at the center of a black hole where the laws of physics, including quantum mechanics, break down. This research project aims to address this gap in knowledge. However, such concepts require incredibly complex mathematical calculations possible only through the use of quantum computers (which we are unable to procure access to, due to their scarcity). Hence, instead of complex equations that involve quantum mechanics, gravitational forces, and quantum corrections, this research project calculates the motion of particles based solely on classical mechanics and gravitational forces. As a result, our findings are theoretical - also in part due to Heisenberg's Uncertainty Principle – nevertheless, they can be useful in the development of general experimental principles.

Keeping in mind that quantum mechanics and the theory of relativity break down at the singularity of a black hole, using classical mechanics allows us to take a step back and simulate the behavior of particles near singularities within a Newtonian framework, where spacetime is constant and motion is predictable. Using classical mechanics instead of quantum mechanics also allows for precise computations of every measurement, unlike the unclear and varying probabilistic predictions from quantum calculations, and the inclusion of Hamiltonian formulation principles provide a direct connection to quantum physics **(pp. 3-4; "Understanding the particle motion equations")**. Our findings provide a clear basis for further understanding of particle behavior within a quantum framework. This project serves as a stepping stone for further future research and understanding of black holes.

Context / General Information

i. Understanding the spherical coordinate system

This simulation uses the three-dimensional coordinate system. Three coordinates are utilized: radial distance (r), polar angle (θ), and azimuthal angle (φ).

- Radial Coordinate (Radius) (r):
    - The straight-line distance from the origin (center of the black hole / singularity) to the particle.
- Polar Angle (Inclination) (θ):
    - The angle formed between the radial position vector and the positive z-axis.
- Azimuthal Angle (Azimuth) (φ):
    - The angle formed in the xy-plane between the positive x-axis and the projection of the radial position vector.

ii. Understanding the particle motion equations

The underlying connection between the concepts of classical mechanics explored in our simulation and the quantum framework in which our project is set lies in the ideas and formulations of Hamiltonian Mechanics - a formulation of classical physics pertaining to motion. Hamilton's equations of motion yield a trajectory in terms of positions and momenta as a function of time and can be used for systems in any set of coordinates, not just the standard Cartesian system, allowing for varying and widespread application. Quantum mechanics makes heavy use of the Hamiltonian and Lagrangian - a similar formulation, from which the Hamiltonian equations can be derived - formulations of physics, which are both based in classical mechanics. For example, the algebraic Heisenberg representation of quantum theory is comparable to the algebraic Hamiltonian representation of classical mechanics, showing how quantum theory evolved from and is related to classical mechanics. Essentially, quantum mechanics is the noncommutative version of classical Hamiltonian mechanics.

In our project, the equations on which the research is structured contains core principles of the Hamiltonian formulations. Specifically, the presence of the momentum terms in the update equations is a characteristic feature of Hamiltonian mechanics - the Hamiltonian formulation is often used in numerical simulations to propagate the state of a system forward in time. Therefore, although the equations were not originally derived from Hamilton's equations, since energy was not our target parameter, they do align with their structures and concepts and distinctly resemble the type of equations that are derived from Hamilton's equations. For instance, the second equation involving the radial momentum almost precisely mirrors Hamilton's equation for the radial coordinate.

Since Hamiltonian formulations are fundamentally a direct connection between classical and quantum physics, the presence of it in our simulation indicates a high degree of validity in possible applications of our findings in a realistic, quantum setting.

Derivations of Equations:

a. The first equation, for radial position, uses basic equations, such as the one for velocity (v = position/time) and derivatives.

$$r_i = \text{ } position\ from\ the\ center\ at\ time\ 'i'$$
$$r_{i-1} = previous\ position$$
$$p_{r_{i-1}} = \text{ } velocity\ at\ time\ i-1$$
$$dt = \text{ } time\ step$$

$$p_r = \frac{dr}{dt}$$
$$\Delta r = p_r * \Delta t$$
$$r_i = r_{i-1} + \Delta r$$
$$r_i = r_{i-1} + (p_r * \Delta t)$$
$$r_i = r_{i-1} + \left(\frac{dr}{dt_{i-1}} * \Delta t\right)$$
$$r_i = r_{i-1} + \left(p_{r_{i-1}} * \Delta t\right)$$

b. The second equation for radial momentum uses the gravitational force equation, along with Newton's 2nd Law. By substituting these two equations and applying them to this scenario, the second equation can be derived.

$$F = G\frac{Mm}{r^2}$$
$$F = ma$$
$$a = \frac{dv}{dt}$$
$$F = m\frac{dv}{dt} = -G\frac{Mm}{r^2}$$
$$\frac{dv}{dt} = -G\frac{M}{r^2}$$
$$\Delta p_r = \frac{dv}{dt} * \Delta t$$
$$p_{r_i} = p_{r_{i-1}} + \Delta p_r$$
$$p_{r_i} = p_{r_{i-1}} + (\frac{dv}{dt} * \Delta t)$$
$$p_{r_i} = p_{r_{i-1}} - G\frac{M}{r^2} * \Delta t$$

c. The third equation, polar angle, can be derived through integration and the basic formulae for angular velocity and angular momentum.

$$\omega = \frac{d\theta}{dt}$$
$$L = I\omega$$
$$\omega = \frac{L}{r^2}$$
$$\int \omega\ dt = \int \frac{L}{r^2} dt$$
$$\theta_t = \int \omega\ dt$$
$$\theta_t = \frac{1}{M}\int \frac{p_\theta}{r^2}\ dt$$

d. The fourth equation, for polar momentum, is found through application of the definition of torque and the force of gravity, angular acceleration, and by substituting these concepts into a larger equation.

$$\tau = rF$$
$$\tau = r(G\frac{Mm}{r^2})$$
$$\tau = I\alpha$$
$$r\left(G\frac{Mm}{r^2}\right) = Mr^r\alpha$$
$$\alpha = -G\frac{M}{r^2}$$
$$\Delta p_\theta = \alpha * \Delta t$$
$$p_{\theta i} = p_{\theta i-1} + \Delta p_\theta$$
$$p_{\theta i} = p_{\theta i-1} - G\frac{Mp_{\theta i-1}}{{r_i}^2}\Delta t$$

e. The fifth equation, for azimuthal angle, is found by first deriving the equation for azimuthal angular velocity, and then substituting it in for azimuthal angle change.

$$p_{\phi} = azimuthal\ angular\ momentum$$
$$L = I\omega\ ; I = Mr^2$$
$$\tau = r(G\frac{Mm}{r^2})$$
$$\omega = \frac{L}{I} = \frac{L}{Mr^2}$$
$$r\left(G\frac{Mm}{r^2}\right) = Mr^r\alpha$$
$$\omega = \frac{p_{\phi_{i-1}}}{Mr_i^{\ 2}}$$
$$\omega = \frac{d\phi}{dt}$$
$$\Delta\phi = \ \omega * \Delta t$$
$$\phi_i = \ \phi_{i-1} + \Delta\phi$$
$$\phi_i = \ \phi_{i-1} + \omega * t_i - t_{i-1}$$
$$\phi_i = \ \phi_{i-1} + \omega * \Delta t$$

f. The sixth equation, azimuthal momentum, is found through the definition of torque and torque in circular orbit and substitution.

$$\tau = r * F$$
$$\tau = r(G\frac{Mm}{r^2})$$
$$\tau = I\alpha\ ; I = Mr^2$$
$$r\left(G\frac{Mm}{r^2}\right) = Mr^r\alpha$$
$$\alpha = \ -G\frac{M}{r^2}$$
$$\Delta p_{\phi} = \ \alpha * \Delta t$$
$$p_{\phi_i} = \ p_{\phi_{i-1}} + \ \Delta p_{\phi}$$
$$p_{\phi_i} = \ p_{\phi_{i-1}} - G\frac{Mp_{\phi_{i-1}}}{r_i^{\ 2}}\Delta t$$

iii. Conservation Laws

The main conservation laws of physics apply in this simulation at the classical physics level and quantum physics level, providing a connection between the two fields.

- Law of Conservation of Mass:
  - The simulation does not include any terms that would change the mass of the particle.
  - Mass is a fundamental property that is constant in this research.
  - It only changes in the presence of nuclear reactions or relativistic speeds, both of which are not included in this simulation.

- Law of Conservation of Charge:
  - The simulation does not account for any change in the charge of the particle.
  - Charge conservation is based on electromagnetism, and no interactions in the simulation would alter the charge of the particle.
  - For charge to change, it must interact with an electromagnetic force field, which is not present in this simulation.

- Law of Conservation of Energy and Momentum:
  - The simulation focuses on the gravitational effects on the particle's motion and behavior. Our research paper discusses the conservation of angular momentum in different directions and the complex dynamics of the particle's motion near a black hole.
  - However, without an explicit energy equation, we cannot confirm energy conservation within the simulation.
  - Our simulation and research focuses primarily on the dynamics of the particle's motion and the conservation of angular momentum, rather than a detailed energy analysis of the particle.

Methods:

Experimental Setup:

The experiment involves creating a microscale black hole simulation in Visual Studio Code with Python as the programming language. Key parameters of the black hole, such as Mass (M), Angular Momentum (J), and Charge (Q), are defined. Meanwhile, precision and simulation duration are controlled by configuring parameters like Time Step (dt), Time Array (t), and Number of Time Steps (N). The simulation employs classical mechanics equations to model particle motion under the influence of black hole gravity.

Simulation code:

```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.widgets import Slider
```

This part of the code imports all the necessary libraries for this simulation. The NumPy library is used to perform various mathematical calculations, whereas MatPlotLib is used for visualized graphing.

```
# Constants
G = 6.6743e-11  # Gravitational constant

# Black Hole Parameters
```

```
M = 1e-8  # Mass of the black hole in kg
J = 1e-9  # Angular momentum of the black hole in kg m^2/s
Q = 1e-19  # Charge of the black hole in C


# Simulation Variables
dt = 1e-17  # Time step (the time interval between each step in the
simulation)
t = np.arange(0, 1e-16, dt)  # Time array (array of values over which
the simulation will run)
N = len(t)  # Number of time steps
```

This code section initializes all the necessary constants and variables needed in the simulation.

```
# Set fixed initial conditions range
r_range = (1e-20, 1e-19) # Radial Position
pr_range = (-1e-20, 1e-20) # Radial Momentum
theta_range = (0, np.pi) # Polar Angle
ptheta_range = (-1e-20, 1e-20) #Polar Momentum
phi_range = (0, 2 * np.pi) # Azimuthal Angle
pphi_range = (-1e-20, 1e-20) # Azimuthal Momentum
```

In the above code segment, the range of values over which each of the parameters run in the simulation are set. This is because some numbers – either extremely high or low - may not be able to be computed by the computer system, leading to an overflow error.

```
def update(val):
    r_0 = r_slider.val
    p_r_0 = pr_slider.val
    theta_0 = theta_slider.val
    p_theta_0 = ptheta_slider.val
    phi_0 = phi_slider.val
    p_phi_0 = pphi_slider.val

    for i in range(1, N):

        # Radial position and momentum
        r[i] = r[i-1] + p_r[i-1] * dt
        p_r[i] = p_r[i-1] - G * M * p_r[i-1] * dt / r[i]**2

        # Polar angle and momentum
        theta_dot = p_theta[i-1] / (M * r[i]**2)
        p_theta[i] = p_theta[i-1] - G * M * p_theta[i-1] * dt /
```

```
         r[i]**2

        # Azimuthal angle and momentum
        phi_dot = p_phi[i-1] / (M * r[i]**2 * np.sin(theta)**2)
        p_phi[i] = p_phi[i-1] - G * M * p_phi[i-1] * dt / r[i]**2
        phi[i] = phi[i-1] + phi_dot * dt

    # Setting up plots for each of particle's parameters
    ax[0, 0].clear()
    ax[0, 0].plot(t, r, color='blue')
    ax[0, 0].set_xlabel('Time (s)')
    ax[0, 0].set_ylabel('r (m)')
    # Same method used to plot the remaining graphs
```

The ‘update’ function updates the simulation based on the slider values for each of the particle’s characteristics. It recalculates the particle’s trajectory and updates the graphs in the outcome. Additionally, this function also is responsible for setting up the plots/graphs for each of the parameters as a function of time.

```
# Create sliders
fig, ax = plt.subplots(3, 2, figsize=(15, 10))
plt.subplots_adjust(left=0.1, top = 0.98, bottom=0.25)
```

These lines are responsible for creating the sliders for each of the initialized parameter values.

```
# Attach sliders to the update function
r_slider.on_changed(update)
pr_slider.on_changed(update)
theta_slider.on_changed(update)

ptheta_slider.on_changed(update)
phi_slider.on_changed(update)
pphi_slider.on_changed(update)
```

This code segment connects the sliders to the ‘update’ function so that a real-time change in the graphs can be seen as the slider values for a parameter are changed.

Outcome:

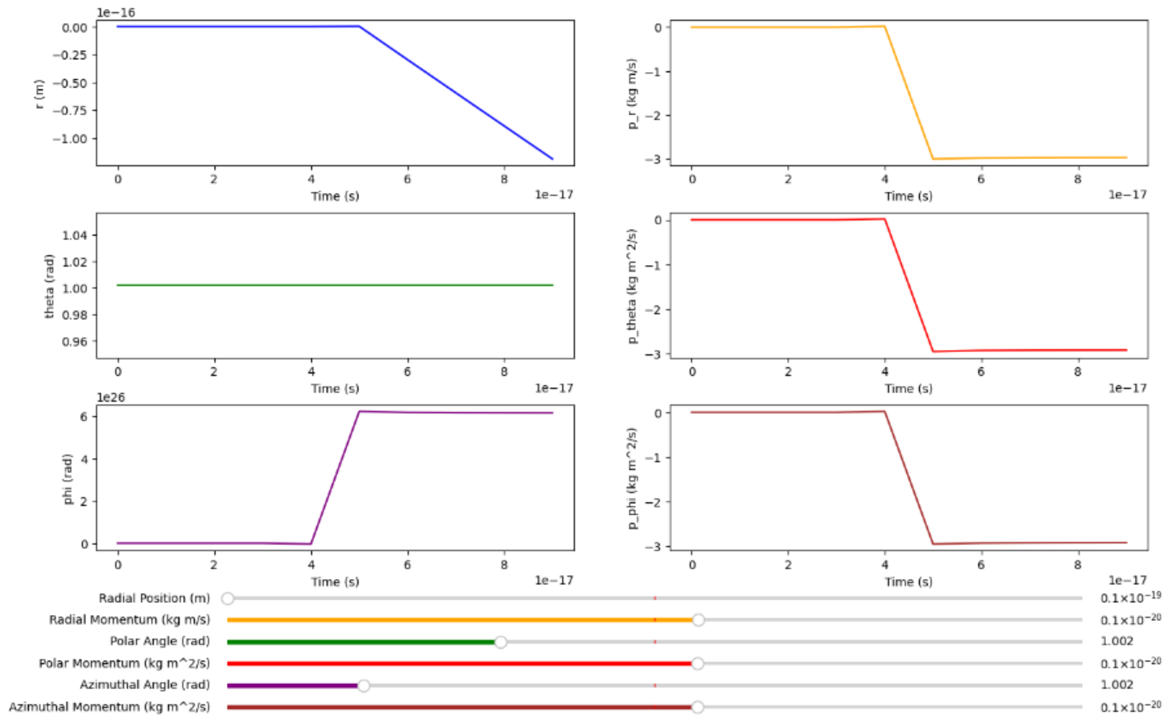

Figure 1: This figure represents the time evolution of key kinematic variables. The left three panels (top to bottom) display radial position 'r' in meters (blue), radial momentum 'p_r' in kg·m/s (yellow), and polar angle θ in radians (green) as functions of time (in seconds). The right three panels (top to bottom) show the time evolution of the azimuthal angle 'ϕ' in radians (purple), the momentum conjugate to θ , 'p_θ' in kg · m^2/s (red), and the momentum conjugate to ϕ, 'p_ϕ' in kg · m^2/s (brown). The slider settings (bottom of panels) control the initial conditions of these variables.

Data Collection:

To study the relationship between two parameters, specific characteristics of the particle's motion were isolated. A particular time point was chosen, and a range of values for the second parameter was explored. Data was collected at equal intervals within this range, and scatter plots were generated. Best-fit curves were applied for detailed analysis. A detailed data collection method is given:

1. Parameter Isolation: Identify 2 parameters (parameter #1 and parameter #2) to explore their relationship. Set a specific value for parameter #1 to isolate its effect. Keep all other sliders at constant.
2. Time Selection: Choose a specific time point in the simulation to collect data on (for best results, choose the time where most dynamic change occurs).
3. Variation Range for parameter #2: Select a range of values to set as initial conditions for parameter #2 from the slider.
4. Data Collection: Run the simulation at specific equal length intervals within the range for parameter #2.
5. Input the corresponding values for parameter #1 and parameter #2 into a graphical analysis application.

6. Graphical Representation: Generate a scatter plot with Parameter #1 on the x-axis and Parameter #2 on the y-axis. Each point represents a specific value for parameter 2 and its corresponding value for parameter 1 at the chosen time point.
7. Apply a best fit curve to determine the relationship between the two parameters.

Data Analysis

I. Radial Position vs. Radial Momentum:

- All variables held constant:
  - Radial Position: 0.1000e-19 meters
  - Polar Angle: 1.002 radians
  - Polar Momentum: 0.1090e-20 kg m^2 / s
  - Azimuthal Angle: 1.002 radians
  - Azimuthal Momentum: 0.1000e-20 kg m^2 / s
- Time Selection: 5.000e-17 seconds
- Slider Ranges (e-20 kg*m/s): [-1.000, -0.8000, -0.6000, -0.4000, -0.2000, 0.0000, 0.2000, 0.4000, 0.6000, 0.8000, 1.000]

Slider Value - Radial Momentum (e-20)	Radial Position (e -19)	Radial Momentum (e - 20)
-1.000	-1.870	0.1440
-0.8000	-1.550	0.2710
-0.6000	-1.167	0.4543
-0.4000	-7.400	0.3210
-0.2000	-0.3400	1.7310
0.0000	0.0000	0.0000
0.2000	0.4500	-1.554
0.4000	0.8300	-0.7350
0.6000	1.210	-0.4273
0.8000	1.630	-0.2500
1.000	2.010	-0.1260

Figure 2: This table holds individual data points collected to summarize the relationship between radial position and momentum for different slider values of radial momentum. This data is used to graph a general parametric plot of radial position vs. radial momentum. The slider value corresponds to the dimensionless radial momentum (in units of 10^{-20} kg·m/s) and the table lists the corresponding radial position (in units of 10^{-20} kg·m/s). Positive slider values result in increasing radial position and decreasing negative radial momenta, while negative slider values result in decreasing radial position and increasing negative radial momenta.

Results (*Figure 3*):

1. The plotted data shows an inverse relationship between Radial Position and Radial Momentum.

2. As the Radial Position increases (moving away from the black hole), the Radial Momentum decreases, and vice versa.

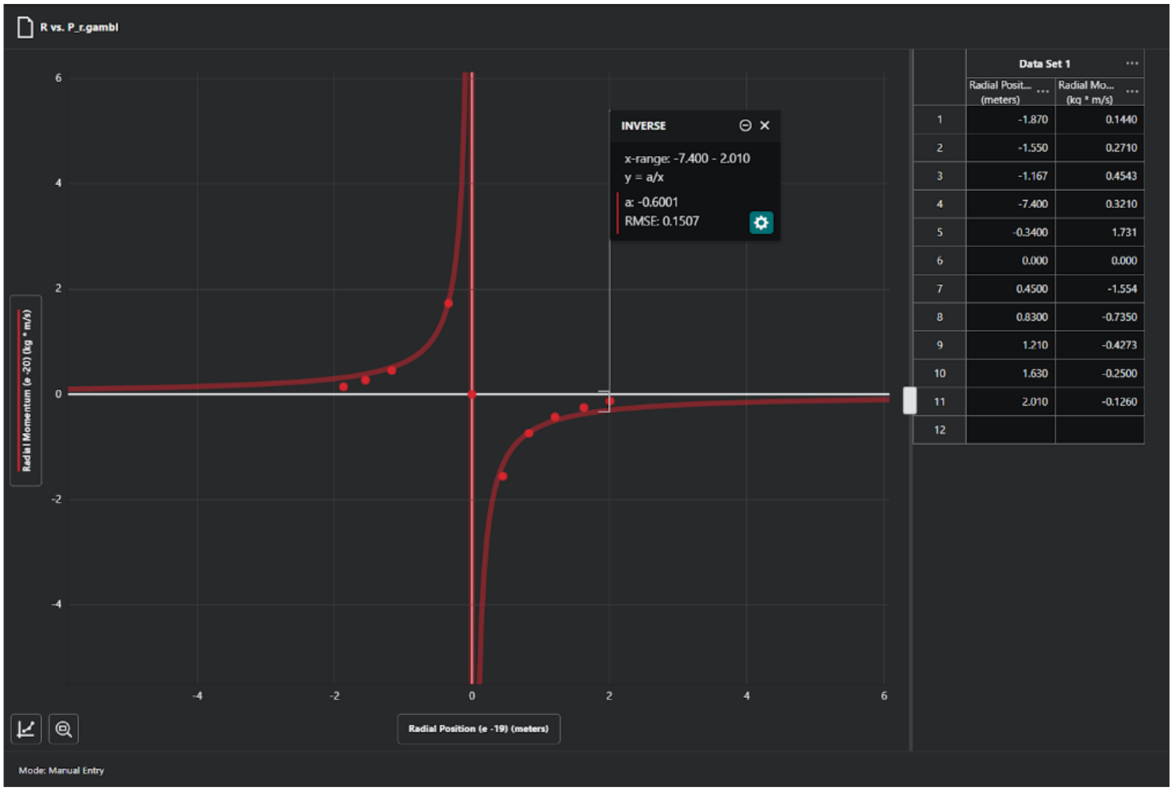

Figure 3: This plot shows the inverse relationship between radial position (x-axis, in units of 10^{-19} meters) and radial momentum (y-axis, in units of 10^{-20} kg·m/s), fitted with a function in the form y=a/x, where a=−0.6001. Data points are represented by red dots, and the fit is represented by the red curve. The data exhibits two branches near a vertical asymptote at x = 0, consistent with the inverse relationship. The goodness of the fit is indicated by a root mean square error (RMSE) of 0.1507, suggesting a reasonably accurate fit of the inverse model to the data.

II. Polar Angle vs. Polar Momentum:

- All variables held constant:
  - Radial Position: 0.1000e-19 meters
  - Radial Momentum: 0.1000e-20 kg*m/s
  - Polar Angle: 1.002 radians
  - Azimuthal Angle: 1.002 radians
  - Azimuthal Momentum: 0.1000e-20 kg * m^2 / s
- Time Selection: 5.000e-17 second
- Slider Ranges (e-20 kg * m^2 / s): [-1.000, -0.8000, -0.6000, -0.4000, -0.2000, 0.0000, 0.2000, 0.4000, 0.6000, 0.8000, 1.000]

Slider Value - Polar Momentum (e-20 kg * m^2 / s)	Polar Angle (radians)	Polar Momentum (e -20 kg * m^2 / s)
-1.000	0.9995	29.74
-0.8000	0.9995	23.80
-0.6000	0.9995	17.83
-0.4000	0.9995	11.92
-0.2000	0.9995	5.960
0.0000	0.9995	0.00060000
0.2000	0.9995	-5.910
0.4000	0.9995	-11.83
0.6000	0.9995	-17.74
0.8000	0.9995	-23.60
1.000	0.9995	-29.50

Figure 4: This table presents the corresponding data for the polar angle (in radians) and polar momentum (in e^{-20} kg·m²/s) as used in Figure 5. The polar angle remains nearly constant at 0.9995 radians for all slider values, while the polar momentum ranges from 29.74 e^{-20} kg·m²/s to -29.50 e^{-20} kg·m²/s. This tabulated data supports the finding that polar momentum does not affect the polar angle significantly, as shown by the constant angle values across a range of momentum values. The data is symmetric around the momentum value when it approaches zero, reinforcing the constant nature of the polar angle.

Results (*Figure 5*):

- The plotted data shows a unique relationship where the Polar Angle remains constant as the Polar Momentum varies.
- Polar Momentum follows an undefined pattern with positive and negative values.

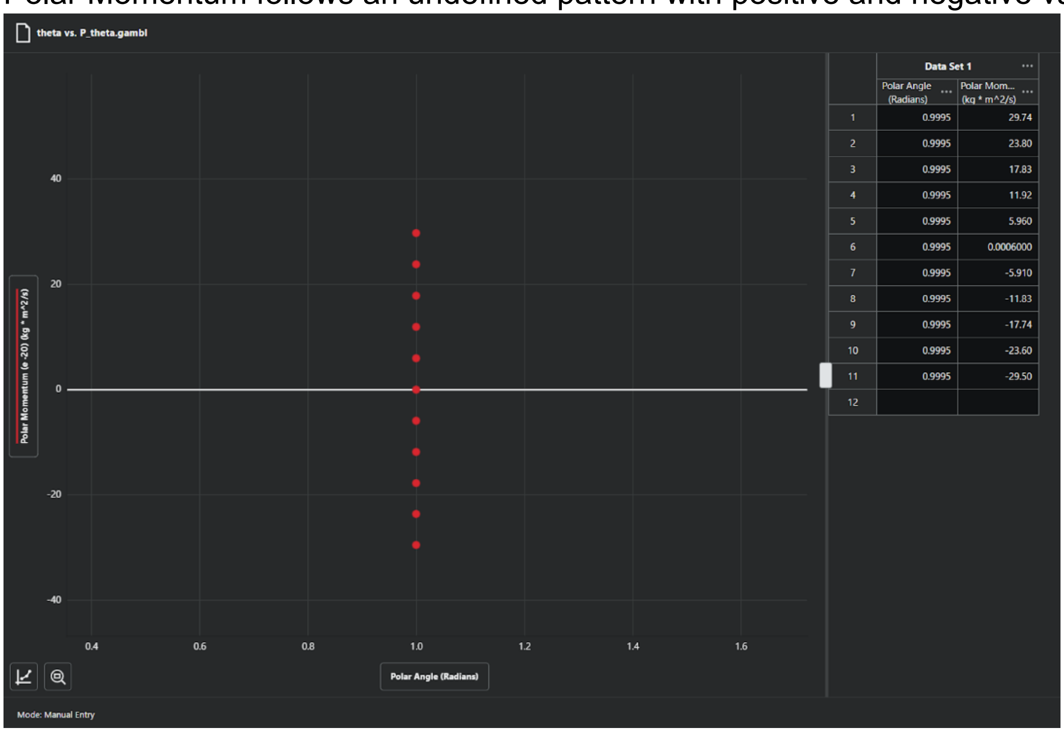

Figure 5: This plot illustrates the relationship between polar momentum (e^{-20} kg·m²/s) and the polar angle (radians). Unlike the trend observed in Figure 3, this graph demonstrates that the polar momentum remains constant as the polar angle remains fixed around 0.9995 radians for all data points. The polar momentum values range from 29.74 e^{-20} kg·m²/s to -29.50 e^{-20} kg·m²/s, indicating a uniform polar angle across varying momentum values. This implies no significant dependence of the polar momentum on the polar angle, suggesting that polar angle remains static all throughout.

III. Azimuthal Angle vs. Azimuthal Momentum:

- All variables held constant:
  - Radial Position: 0.1000e-19 meters
  - Radial Momentum: 0.1000e-20 kg * m/s
  - Polar Angle: 1.002 radians
  - Polar Momentum: 0.1000e-20 kg * m^2 / s
  - Azimuthal Angle: 1.002 radians
- Time Selection: 5.000e-17 second
- Slider Ranges (e-20 kg * m^2 / s): [-1.000, -0.8000, -0.6000, -0.4000, -0.2000, 0.0000, 0.2000, 0.4000, 0.6000, 0.8000, 1.000]

Slider Value - Polar Momentum (e-20 kg * m^2 / s)	Azimuthal Angle (e27 radians)	Azimuthal Momentum (e -20 kg * m^2 / s)
-1.000	-6.250	29.80
-0.8000	-5.000	23.80
-0.6000	-3.740	17.85
-0.4000	-2.490	11.94
-0.2000	-1.260	5.970
0.0000	0.9971	0.0002000
0.2000	1.260	-5.910
0.4000	2.530	-11.88
0.6000	3.780	-17.68
0.8000	5.070	-23.70
1.000	6.310	-29.50

Figure 6: This table provides the corresponding data for the azimuthal angle (in radians) and azimuthal momentum (in e^{-20} kg·m²/s) as used in Figure 7. It presents values ranging from -6.250 radians to 6.310 radians as corresponding to an azimuthal momentum of 29.80 e^{-20} kg·m²/s to -29.50 e^{-20} kg·m²/s. Notably, the azimuthal momentum decreases linearly with increasing azimuthal angle, consistent with the graphical trend observed in Figure 7.

Results (*Figure 7*):

- There is a linear relationship between Azimuthal Angle and Azimuthal Momentum.
- The line of best fit has a negative slope, indicating a proportional relationship.

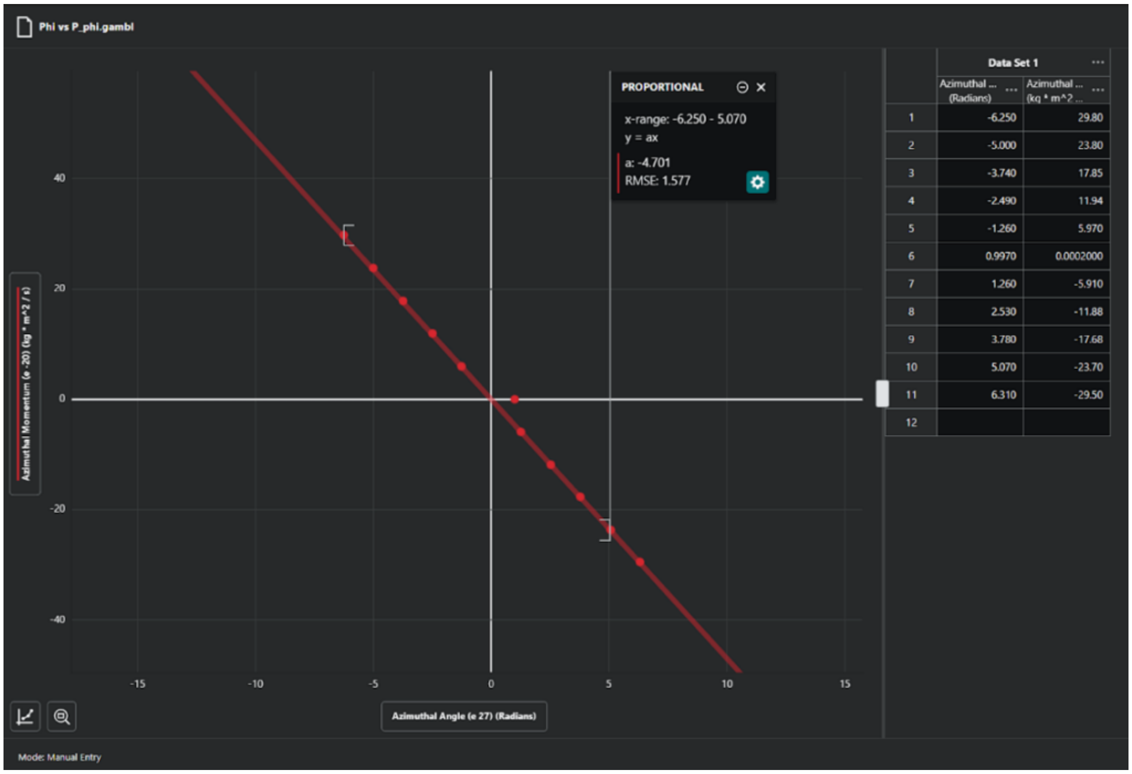

Figure 7: This plot represents the relationship between the azimuthal momentum (e^{-20} kg·m²/s) and the azimuthal angle (e^{27} radians). The data points follow a clear negative linear trend, indicating that as the azimuthal angle increases, the azimuthal momentum decreases. A linear proportional fit is applied to the data, yielding a slope a=−4.701 with a root mean square error (RMSE) of 1.577, showing the precision of the fit. The linear trend remains across the full range of azimuthal angles from approximately -6.25 to 6.31 radians. The data points, as collected from Figure 6, are symmetrical and evenly distributed across the range.

Analysis

I. Radial Position vs. Radial Momentum: As the particle moves away from the black hole, its radial momentum decreases to conserve angular momentum. Conversely, as the particle moves closer to the black hole, its radial momentum increases. This inverse relationship is consistent with the conservation of angular momentum and the effects of gravitational forces, which accelerate the particle as it moves closer to the black hole and decelerate it as it moves away.

II. Polar Angle vs. Polar Momentum: The constant polar angle suggests that the particle remains on a fixed plane throughout the simulation. The linear relationship with polar momentum implies a direct relationship. This suggests that changes in the polar momentum do not affect the particle's inclination relative to the equatorial plane within the scope of this simulation.

III. Azimuthal Angle vs. Azimuthal Momentum: This linear relationship suggests that the particle's azimuthal motion is proportional to its azimuthal momentum, following a simple rotational motion. The negative slope indicates that as the azimuthal angle increases, the azimuthal momentum decreases, and vice versa. This behavior aligns with the conservation of angular momentum in the azimuthal direction/plane in a spherical coordinate. It also shows the changes in the rate of rotation of the particle around the z-axis.

Observations and Inferences

- The observed behaviors in Radial Position vs. Radial Momentum and Azimuthal Angle vs. Azimuthal Momentum align with the conservation of angular momentum in different directions.
- The constant Polar Angle while Polar Momentum varies suggests that the particle remains on a fixed plane throughout the simulation. We can infer that the polar angle does not fluctuate since the polar momentum is stabilizing the particles motion, preventing changes to the polar angle.
- The unique behaviors observed in the Polar Angle vs. Polar Momentum relationship indicate complex dynamics that may be specific to this research experimental simulation setup.

From the simulation and analysis, we can infer that the motion of a particle in the vicinity of a black hole is significantly influenced by the gravitational field. The inverse relationship between radial position and radial momentum suggests that as a particle falls into the black hole, its speed increases due to gravitational acceleration. The constant polar angle implies that the particle's inclination remains unchanged during the simulation, which could be due to the symmetrical nature of the gravitational field around the black hole. The linear relationship of polar momentum indicates a conservation of momentum in the absence of external torques affecting the particle's motion in the polar direction.

Conducting Quantum Corrections

While classical dynamics provides a foundation for understanding macroscopic phenomena, quantum dynamics focuses on the microscopic. To render the simulation findings valid in this microscopic world, concepts of quantum mechanics must be integrated into the classical framework of the research. Multiple theoretical methods of quantum correction can be applied in this situation.

Particles near a black hole experience continuous oscillations due to the strong gravitational field presence. Calculating its movement can be done using the framework of a harmonic oscillator potential, $E = \frac{1}{2}mu^2 + \frac{1}{2}kx^2$ where the black hole's gravity acts as a "restoring force," confining the particle's motion around an equilibrium position. However, in conceptualizing this behavior, the wave-particle duality must be taken into consideration. Therefore, wave functions (Ψ(r)) must be included in our classical motion equations, where "r" represents the distance from the black hole. |Ψ(r)|^2 would then indicate the probability of finding the particle at a specific distance "r", with a higher value squared indicating a higher probability area. This can also be calculated in terms of Hilbert space formalism, in which wave functions are represented as vectors, so the probability density mentioned represents the inner product of the wave function vector with itself. To incorporate these complex features of quantum physics, mathematical operators must be used - the position operator ($\hat{x}$) and momentum operator ($\hat{p}$) - and integrated into the classical motion equations. Subsequently, the plotting of different wave functions per region of the black hole would result in waves with varying amplitudes, exhibiting the different probabilities of the particle's position around the black hole. A high peak suggests a greater probability, while a greater spread suggests a wider range of outcomes, resulting in a lower probability.

Such wave functions also obey Schrödinger's equation, encapsulating the system's properties. Additionally, much of Schrödinger's equation incorporates Hamiltonian operators $\hat{H}\Psi = E\Psi$, which, as discussed previously, bridges the gap between Newtonian and quantum physics. In the framework of Hilbert spaces, operators like the Hamiltonian act on wave functions, transforming them into other wave functions. By solving Schrödinger's equation, considering the gravitational potential generated by the black hole, it would then be possible to model and identify the quantized energy levels around the black hole – another key characteristic of quantum mechanics. The quantized energy levels resulting from the model would then prove and support the hypothesis, suggesting that the particle holds discrete energies, rather than a continuous spectrum. However, it's important to note that there is no one specific wave function that would be able to model the probability for the entire black hole, due to the shape of the gravitational potential around black holes, which influences the wave functions and the corresponding energy levels, resulting in regions with different corresponding energy levels.

Another key phenomenon to take into consideration while applying quantum corrections is wave packet spreading. As mentioned previously, the Heisenberg Uncertainty Principle renders it impossible to precisely determine both the position and momentum of a particle simultaneously, a concept in conflict with the data collection in this study. However, this uncertainty could be manifested as the phenomenon of wave packet spreading in the particle's wave function, where narrowing the particle's position probability distribution leads to increased uncertainty in its momentum, and vice versa.
In order to take this into account, time-dependent Hamiltonian operators (for example, the time-dependent Schrödinger equation: $i\hbar \frac{d}{dt}|\Psi(t)\rangle = \hat{H}(t)|\Psi(t)\rangle$) must be included in the wave function. It would then be possible to see how the uncertainty in the position of the particle is related to the uncertainty in its momentum by incorporating a dispersion relation into the equations. And incorporating Schwarzschild Metric corrections, which not only is analogous to the classical Newtonian theory of gravity but also takes into account Einstein's theory of general relativity as well as the field equations, would give outcomes that model a Schwarzschild black hole.

Due to the absence of a quantum computer, such quantum calculations could not be processed in this study. Therefore, a more practical and semi-classical approach was adopted, where particle motion is calculated with classical equations, but quantum corrections are applied for only specific properties. This particular application also allows for a focus on particle dynamics and motion - the target subjects for analysis in this study. Although actual numbers and solutions are not provided, the practical implementation of the key quantum mechanics theories captures the essential quantum effects with computational feasibility. Additionally, the connections between the two realms of physics - classical and quantum - made in this study contribute to the understanding and possible development of the “theory of everything”.

Hypothesis Evaluation

The hypothesis in this research project stated that the motion of the particle near the microscale black hole is composed of continuous and periodic oscillations in multiple dimensions, involving translational and angular dynamics.

The extent to which the hypothesis was correct:

- The motion exhibits continuous changes in parameters over time, reflecting the influence of gravitational forces.
- The periodic nature is evident in the repetitive patterns observed in the relationships between radial position and radial momentum, as well as azimuthal angle and azimuthal momentum.
- The simulation involves both translational (radial position, radial momentum) and angular (polar angle, polar momentum, azimuthal angle, azimuthal momentum) dynamics, confirming motion in multiple dimensions.
- The observed behaviors, such as the inverse relationship in Radial Position vs. Radial Momentum and the proportional relationship in Azimuthal Angle vs. Azimuthal Momentum, suggest a dynamic equilibrium.
- Despite the complexity in the findings, there are indications of stability and repeatability in trajectories, supporting the notion of a dynamic equilibrium.

This research project investigates the gravitational effects of a microscale black hole on the motion of a particle, addressing the overarching questions of how these effects shape the particle's dynamics and how the characteristics of its motion interrelate. The observed relationships between parameters, such as the inverse correlation in Radial Position vs. Radial Momentum and the proportional connection in Azimuthal Angle vs. Azimuthal Momentum, indicate a dynamic equilibrium. However, stable and repeatable trajectories are still visible, suggesting a gravitational influence that stabilizes the particle's motion over time.
It is also important to note that classical mechanics can provide insights into the motion of particles near a black hole, despite its quantum limitations. This is supported by the simulation. The classical equations used in the simulation have successfully demonstrated predictable relationships between the parameters of particle motion. However, it is important to note that these findings are theoretical and based on a simplified model that does not account for the complex interactions present in a real quantum gravitational environment.

Conclusion

The gravitational effect of a black hole has a significant impact on a particle's motion, as demonstrated by the simulation. The gravitational pull of the black hole dictates the particle's trajectory, influencing both its translational motion (movement from one point to another) and its angular motion (rotation around a point). Specifically, the simulation results indicate that as a particle moves closer to the black hole, its radial momentum increases due to the gravitational acceleration, while its radial position decreases. This inverse relationship is a direct consequence of the gravitational force exerted by the black hole, which intensifies as the distance to the singularity decreases. Moreover, the gravitational field of the black hole affects the particle's angular momentum. For instance, the azimuthal angle and momentum exhibit a linear relationship with a negative slope, suggesting that as the particle rotates around the black hole, its momentum decreases with increasing angle, consistent with the conservation of angular momentum in a central force field like that of a black hole.

The characteristics of a particle's motion are intricately linked, reflecting the complex dynamics within the gravitational field of a black hole. The simulation reveals that changes in one aspect of the particle's motion can influence others due to the interconnected nature of the motion parameters. For example, the radial position and momentum are inversely related, indicating that as the particle's distance from the black hole changes, its speed adjusts accordingly to conserve angular momentum. The polar angle's constancy across varying polar momentum values suggests that the particle's inclination relative to the equatorial plane remains stable, which could be indicative of the symmetrical nature of the gravitational field exerted by the black hole in this simulation. This stability in the polar angle, despite changes in polar momentum, points to a dynamic equilibrium where the particle maintains its plane of motion. In the azimuthal dimension, the linear relationship between the azimuthal angle and momentum, with a negative slope, implies that the particle's rotational speed around the black hole is directly influenced by its angular momentum.

This relationship is again a manifestation of the conservation of angular momentum, where the particle's motion adjusts to maintain equilibrium within the gravitational influence of the black hole. In summary, the motion characteristics of a particle near a black hole—radial position, radial momentum, polar angle, polar momentum, azimuthal angle, and azimuthal momentum—are not independent, exhibiting a dynamic equilibrium shaped by the gravitational forces of the black hole. This simulation underscores the complex interplay between these parameters, providing insights into the particle's motion in a strong gravitational field.

The stability observed in the polar angle of the particle’s motion suggests that a confinement of the motion within at least one plane has attributed to the symmetrical nature of the gravitational field surrounding the black hole. Symmetrical black holes also indicate a symmetric collapse of massive stars, leading to a uniform distribution of mass and energy around the star itself.

This symmetry also extends to the accretion process, effecting the distribution of matter within the black hole and facilitating the formation of stable trajectories. The stability of these trajectories also allow matter to consistently orbit the black hole, leading to the formation and maintenance of accretion disks over long periods of time. This sustained accretion process contributes to the growth of the black hole’s mass which in turn strengthens its gravitational pull. The energy emitted from the accretion process plays a crucial role in triggering star formation in nearby regions by heating surrounding gas, dust, and ionizing gas particles.

The conservation of angular momentum, both radially and azimuthally, governs the rotational motion of matter in the accretion disk. If angular momentum is conserved, the disk forms efficiently, allowing for the spiraling of matter towards the black hole and generation of electromagnetic radiation through processes like synchrotron radiation. However, if angular momentum is no conserved, the disk's formation is inefficient, resulting in radiation emission and instability in the disk's motion.

The symmetrical geometry of spacetime near black holes, also predicted by general relativity, aligns with the symmetries observed in gravitational fields and gravitational waves emitted by black hole systems. Understanding these symmetries provides insights into black hole formation, the behavior of matter in extreme conditions, and the overall structure and evolution of galaxies.

Significance of the findings

The findings from this study contribute multiple significant insights into the inner workings of black holes and the nature of quantum physics in terms of space-time as a whole.

I. Black Hole Information Paradox

The black hole information paradox is a widely debated black hole phenomenon that, essentially, points out a conflict between two apparently true facts - information is always conserved, but when it enters a black hole, it seems to be destroyed. However, this study's findings seem to contradict this concept. The "information" in this study - angle and momentum of the particle - stays conserved as radial position decreases to an extent where the statistically significant majority of its probabilities - as in the hypothetical wave function described previously - lies within the event horizon, calculated by the equation

$$R_s = 2GM/c^2$$

where

$$G = 6.6743\text{e-}11\ N{\cdot}m^2/kg^2$$
$$M = 1\text{e}8\ kg$$
$$c = 299{,}792{,}458\ m/s$$

The resultant value of **1.485e-19 m** is included in the range of radial positions tested. Therefore, these findings contribute significant proof to the idea that information is not destroyed within a black hole and could prove instrumental in solving the black hole information paradox.

II. Testing theories of fundamental physics

The simulation in this study can potentially serve as tests of various theories of fundamental physics, including general relativity, quantum gravity, and potential modifications to these theories. Deviations from theoretical predictions could indicate new constraints on gravitational theories or even a new theory. For example, one of the key ideas of general relativity is the bending of light in the presence of a strong gravitational force. By setting the particle in the simulation to have similar properties as a photon, one could theoretically test this aspect of general relativity within quantum constraints.

Based on the data collected, the trajectory of the particle could be depicted with a spiral (*Figure 8)*. This observation aligns with the understanding that the curvature of spacetime primarily depends on the radial position and time. Terms involving angles remain constant and do not reveal information about the curvature of spacetime. Therefore, by focusing on the relationship between radial position and time, the data is plotted for radial position against time, revealing an exponential decay relationship (*Figure*). This trend corresponds to the expected behavior in curved spacetime, validating the predictions of general relativity.

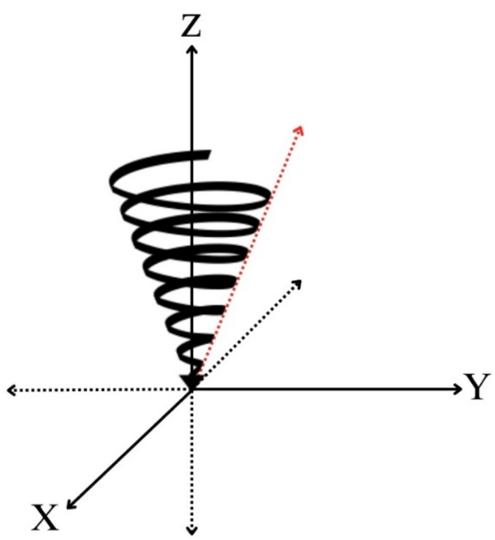

Figure 8: This diagram depicts the 3-dimensional motion of a charged particle spiraling inwards in a helical path along the z-axis in the presence of a uniform gravitational or magnetic field. The z-axis shows the direction of axial motion while the x and y-axes represent circular motion. The pitch of the helix indicates a gradual decrease along the z-axis.

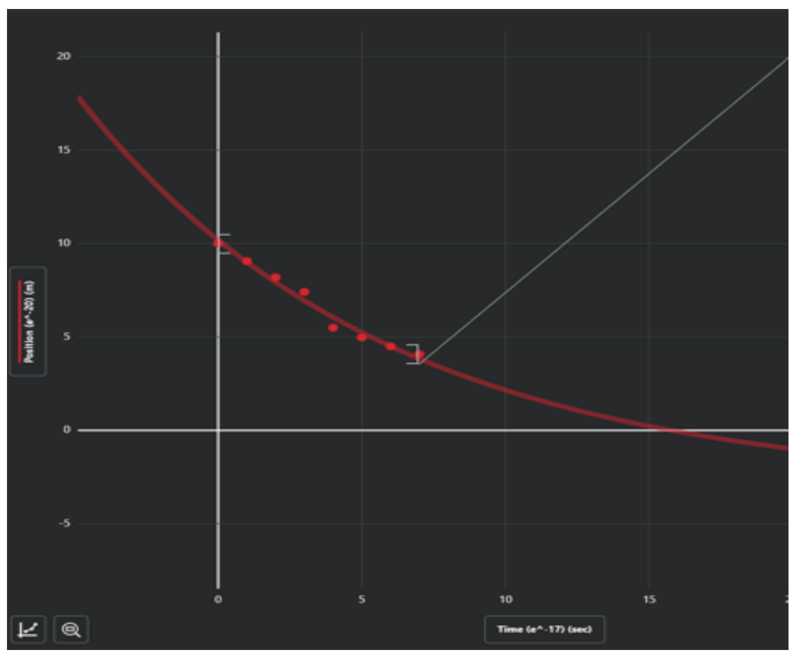

Figure 9: The plot shows the relationship between the particle's radial position (y-axis, e^{-20}meters) and time (x-axis, seconds) near a system under gravitational influence. The graph reveals a rapid decline in position during the initial time period, followed by a slower rate of change as the time increases. The nonlinear decay follows a negative exponential trend where the data points' alignment with the curve indicates agreement with the theoretical model for general relativity.

This would not have been the case for a linear trend since that would not have been consistent with the expected increase in momentum over time. Therefore, the data and findings support the principles of general relativity and provide evidence of curved spacetime, offering valuable insights into the fundamentals of black hole's general relativity.

III. Understanding the interplay between classical and quantum physics

For years, physicists have strived to develop a unified theory - the "theory of everything" - that encompasses both classical and quantum physics, such as string or loop theory **(pp. 21; “Comparing with existing theories”)**.

Such a theory would provide a comprehensive framework for understanding the universe at all scales, from the smallest particles to the largest structures. Achieving this goal could lead to significant insights into the nature of reality and the fundamental laws governing the universe.

Therefore, the experimental method used in this study has profound implications. Since the data collection was conducted from the products of classical calculations, resulting in the need for quantum corrections in order to accurately model a realistic black hole, we are essentially shifting between the two fields, exemplifying the ways in which these two fields are compatible and possibly contributing to the formation of the "theory of everything".

IV. Development of Quantum Technologies

Quantum technologies, such as quantum computing, quantum cryptography, and quantum sensors, are essential to the experimentation and understanding of quantum physics. This study advances the development of such technologies in several ways. First, the use of technology - the black hole simulation - to conduct this experiment contributes to the growing usage of simulation-based experiments in the field of physics, paving the way for the possible experimentation of a wide variety of subjects, many of which would be impossible to conduct physically.

Additionally, understanding the classical limit of quantum mechanics and the quantum behavior of classical systems - one of the focuses of this study, as described above - can help in the design and optimization of these technologies, potentially leading to new breakthroughs in computing, communication, and sensing.

Comparing with existing theories

One of the most prominent unifying theories is the Loop Quantum Gravity Theory. Both this study and LQG involve the concept of quantization, but in different contexts. LQG proposes that spacetime itself is quantized, composed of discrete fundamental units or loops. Similarly, this study suggests that the motion of particles near black holes may exhibit quantized behavior due to the previously described application of quantum corrections to the classical mechanics equations. This alignment with quantized states supports the underlying principles of LQG.

LQG proposes that spacetime itself is not smooth but has a fundamental “grainy” structure due to the discrete loops, similar to how some materials are made of tiny atoms. This graininess is predicted to be at the Planck scale, which is incredibly small. According to LQG, the curvature of spacetime, which is how gravity works in Einstein's theory, would be affected by this quantized structure. In this study, the relationships between the particle's position and momentum, are shown to be inverse relationships. These relationships indicate the particle is confined to specific states rather than existing consistently across a range of values, meaning its motion isn't completely free. This confinement could be due to the influence of the black hole's gravity, and the specific states could be related to the particle's discrete energy.

Apart from that, both LQG and this study serve as bridges between classical and quantum physics. In LQG, classical general relativity is extended to incorporate quantum principles, providing a quantum description of gravity. This bridges the gap between the classical understanding of gravity as described by Einstein's equations and the quantum nature of the universe. Similarly, this study utilizes classical mechanics equations to model particle motion near black holes but introduces quantum corrections to account for the effects of quantum gravity. By incorporating quantum corrections into classical simulations, this study explores how classical and quantum physics intersect in the context of gravitational interactions. Therefore, both approaches bring together classical and quantum aspects, offering insights into the fundamental nature of spacetime and the behavior of particles near black holes.

It is also important to note, however, that although LQG is characterized by its quantum geometric structures - such as loops - this study does not explicitly incorporate such features. Instead, it focuses on applying quantum corrections to classical mechanics equations without introducing quantum geometric elements like loops. Furthermore, due to the different approaches, the distinct mathematical computations and computational methodologies vary as well.

Further Exploration and Quantum Predictions

The equations used in this simulation – reminiscent of Hamiltonian equations – provided for the particle's motion near a black hole, which are based on classical mechanics, can offer insights into the potential connections between classical and quantum physics. The Hamiltonian formalism **(pp. 3-4; “Understanding the particle motion equations”)**., a central concept in classical mechanics, can be extended to quantum mechanics through the process of quantization.

The classical Hamiltonian, which describes the total energy of a system in terms of position and momentum, could be replaced by an operator in the quantum Hamiltonian. This operator will represent the observable associated with the total energy of the system in quantum mechanics. If this is done, the continuous and periodic oscillations observed in the classical motion of the particle could be indicative of quantized energy levels in a quantum description. This would then suggest that the particle's energy near a black hole may be constrained to discrete levels, leading to quantized behavior in its motion.

Additionally, by examining the classical equations, more speculations can be provided about the implications of the particle’s motion in a more quantum framework. For instance, the inverse relationship between radial position and radial momentum, as well as the linear relationship between polar momentum and polar angle, could hint at potential uncertainty relations in quantum mechanics. These relationships might lead to the formulation of generalized uncertainty principles that govern the behavior of particles near black holes at a quantum level.

This uncertainty could be manifested as fluctuations and probabilistic behavior in the particle's motion near a black hole, which contrasts with the deterministic nature of classical mechanics.

However, hypothetically, the behavior of particles near a black hole would undergo significant changes if the Heisenberg Uncertainty Principle did not apply. If this principle were to be disregarded, it would imply that the position and momentum of a particle could be precisely determined at the quantum level, contrary to the inherent uncertainty postulated by the principle. In the context of this simulation, the absence of the Heisenberg Uncertainty Principle would lead to a fundamental shift in our understanding of the particle's behavior.

Specifically, the precise determination of the particle's position and momentum would enable a more deterministic description of its motion. This would result in a departure from the probabilistic nature of quantum physics, allowing for exact predictions of the particle's trajectory and behavior near the black hole.

Furthermore, general understanding of the particle's energy levels and quantized behavior would be impacted. In quantum physics, the uncertainty principle is closely linked to the concept of quantization, where certain observables, such as energy, are constrained to discrete levels. Without this principle, the energy levels and quantized behavior of the particle near the black hole would be subject to reevaluation, potentially leading to a more continuous energy state.

Reflections

For future research, the inclusion of more complex equations with quantum corrections would be crucial in observing more precise dynamic motions of the particle, as they wholly apply to a realistic black hole.

Exploration of unique conditions leading to observed behaviors such as the change in position, angle, and momenta values would be beneficial in providing insights as to the motion of the particle in the spherical coordinate system. Apart from that, understanding and gaining validation through theoretical comparisons and theoretical predictions will help improve overall grasp and representation of these concepts.

The research contributes valuable insights into the gravitational effects on microscale black hole-particle interactions, laying the foundation for further exploration and understanding of the underlying physical processes.

Author Contributions

The authors confirm contributions to the paper as follows: N.O. and H.P. planned the experiment, interpreted the results, and wrote the manuscript. Both provided critical feedback and helped shape the research, analysis, and paper.

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