Building Propulsion Systems for Rockets and, Measurement and Analysis of Heights Attained After Launch

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Abstract: Rocket uses the principle of variable mass system and Newton's laws of motion. In this study three propulsion systems were designed, the propulsions systems were air pressure based, CO2 based and Chemical based. The aim of this study was to calculate the theoretical height attained by the rockets under lab conditions and compare it with the actual height attained, for this purpose, attained height (H_a) vs estimated height (H_e) ratio is calculated, and is termed as the efficiency of the rocket propulsion system. A total of five launches for each propulsion system is conducted and the efficiencies are calculated as follows, for air-based propulsion system 0.56 +/- 0.12, for CO2 based propulsion system 0.30 +/- 0.09 and for chemical-based propulsion system it is 0.30 +/- 0.07.

Keywords: Propulsion system, rocket launch, estimated height, attained height, propulsion efficiency.

1. Introduction

The first rocket was launched on 16th March 1926 by Dr Goddard Milton Lehman. It was the first liquid-fueled rocket, about 3-meter long in height, used liquid oxygen and petrol for combustion. The rocket only flew 12 meters in height after the launch. This launch was the step stone for the Saturn V rocket's launch, which used the same technology to carry humans to the Moon [1]. The first rocket to reach space in vacuum(space) was the world's first artificial satellite sent to space by then the Soviet Union, on October 4th, 1957. The satellite was known as Sputnik - I. The satellite was about 84 kg 3in weight and had a spherical design. Two months later, Sputnik - II reached space with a living passenger, a dog named Laika, the spacecraft orbited Earth for a few hours. The third rocket to reach space was Explorer - I, launched on 31st January 1958 by the United States. Explorer - 1 had made a very important discovery when launched into space, which is now known as Van Allen Radiation Belts, a very important discovery for communication and transmissions of signals [2]. The first man in Space was Yuri Gagarin. On April 12th, 1961, space became the domain of humans with the launch of cosmonaut Yuri Gagarin. His spacecraft was named Vostok - 1. The flight lasted for about 2 hours, and had orbited Earth at a height of about 315 km. This was a crucial step for the development of space science. The next giant leap was about to come in another few years. On July 20th, 1969, Neil Armstrong was the first human to step on an astronomical object other than Earth, it was the Moon. The rocket used for this launch was Saturn V. Since then, there have been numerous additions in space exploration, from Voyager - 1 and 2, to Chandrayaan 1, 2 and 3

A rocket is a mechanical device used to escape Earth's gravitational field, or gravitational field of any astronomical object in general, and it can travel through space. A rocket has many compartments, these are (a) a combustion chamber, in which the propellant(fuel) are burned and

converted into hot gaseous, (b) a nozzle to accelerate the high gas propellant-burned-fuel, (c) propellant container, which stores the container, (d) a structure to support and protect all these parts, and may have (e) a chamber at the top of the supporting structure, which carries astronauts.

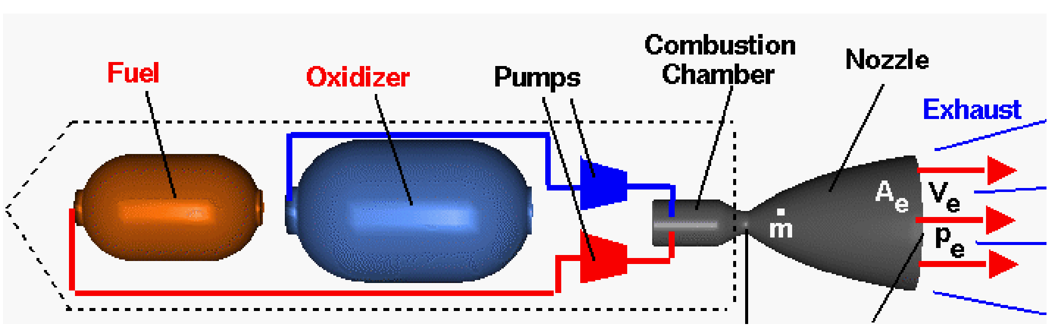


Figure No 1 [3]: A simple model of components of rocket

The most important part of the rocket is its propulsion system, which powers the rocket to escape the gravitational field of the astronomical body. The propulsion system consists of the combustion chamber, propellant container and the nozzle [see figure 1]

There are a few different types of rocket propulsions available for rockets, which are (i) Liquid-fuel, (ii) Solid-fuel, (iii) Cold-gas propulsion and (iv) Ion based propulsion. Depending on the need of the space mission, one chooses the type of propulsion system.

A liquid fuel rocket uses fuels such as liquid-oxygen or liquid-nitrogen. This fuel gets burned to generate energy. To burn the fuel, an oxidizer is required, which provides the oxidation to the fuel, for instance liquid oxygen. A system of pumps to carry the fuels and the oxidizers to mix it together.

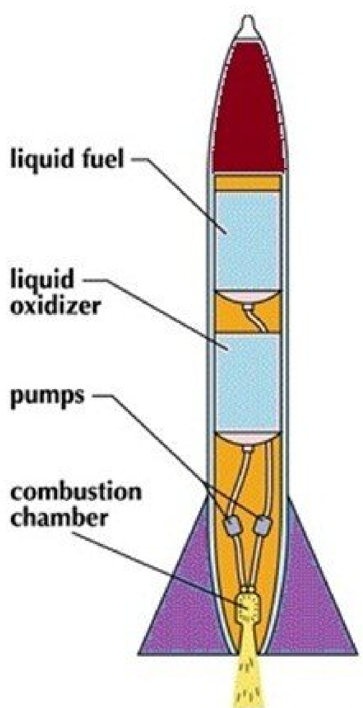


Figure No 2 [4]: Liquid Propellant Propulsion System

A combustion chamber where the liquid and oxidisers mix and burn, later it get out from the exhausts with a high speed [see figure 2]. A solid fuel, such as ammonium perchlorate (AP), ammonium nitrate, or ammonium dinitramide is used in a solid-fuel propulsion system. An igniter to combust the propellant surface, the hole in the middle of the propellant acts as a combustion chamber, which exists from the nozzle at a very high speed [see figure 3].

2.0 Methodology

This section discusses the methodology used to conduct the research.

2.1 Aim of the Study

To measure the efficiency of different propulsion systems

2.2 Research Design

Attained height is compared with the theoretically estimated height, the ratio is the propulsion system efficiency.

2.3 Data Collection Procedure

For each propulsion system five launches are made and the attained height is measured. The observed attained height of the rocket is termed as H_a, where as, the estimated height is termed as H_e, and the ratio of the attained height vs the estimated height is defined as the efficiency of the

rocket, mathematically, efficiency = height attained/height estimated = H_a/H_e. This approach is a simplistic way to describe efficiency of a rocket, and thus, in this paper, this is how the efficiency is described.

3.0 Discussion

3.1 Newton's Second Law of Motion

A body will be in the state of rest or motion when observed from an inertial frame of reference unless and until an external unbalance force exerts on the body.

Mathematically, force is defined as the rate of change of momentum of the body:

$$F = \frac{dp}{dt}$$

$$F = \frac{d(mv)}{dt} \quad \text{. . . (substituting p = mv)}$$

$$F = m\frac{dv}{dt} + v\frac{dm}{dt}$$

The second term is related to mass varying systems, such as a rocket. If v = constant, $\frac{dv}{dt} = 0$, imply, for an object moving with constant velocity, $F = \frac{dm}{dt}$.

3.2 Newton's Third Law of Motion

Every action(a force) will have an equal and opposite reaction(reactive force).

The momentum of a system is unchanged unless and until an external unbalanced force acts on it. Mathematically, this can be modeled as, momentum of the system before is same as momentum of the system after.

$$P_{before} = P_{after}$$

Thus, the momentum of the gas emitted out of the nozzle by burning will be equal to the momentum of the rocket in the vertically upward direction, mathematically,

$$v_{rocket} = (m_{fuel}/m_{rocket}) \times v_{fuel}$$

3.3 Conservation of Energy

The energy of a system will be unchanged, due to any internal interactions of the system, only an external unbalanced force when work on the system provides or removes energy from the systems.

3.4 Energy and Work Done Theorem

Any work done on the system gets converted to energy of the system and vica versa. For example, if an initial momentum of velocity v is given to a rocket by work done on the rocket, then the rocket will be launched in vertically upward direction to a height such that, the kinetic energy at initial is same as that of the potential energy at final.

3.5 Efficiency of Engines

Calculating efficiency of rockets is a complicated process [5]. Efficiency of rockets are calculated as:

$$\eta = \eta_p \eta_c$$

where,

η_p - propulsive efficiency, which is related to the kinetic energy of the exhaust.

η_c - combustion efficiency, which is related to the combustion efficiency of the fuel

η - effective efficiency

The efficiency of rockets can be anywhere from 30 to 65 percent at maximum.

3.6 Material Used

3.6.1 Air-Based Rocket Propulsion System:

An air pump is used to create air pressure in the air pressure propulsion component. The air is stored initially in a PVC pipe system designed to store and hold the air pressure when required. One of the terminals of the pump is connected to the PVC pipe system, whereas the other end is connected to a solenoid, which is controlled by a 7.2 V battery. The designed launchpad is shown in figure no 4.

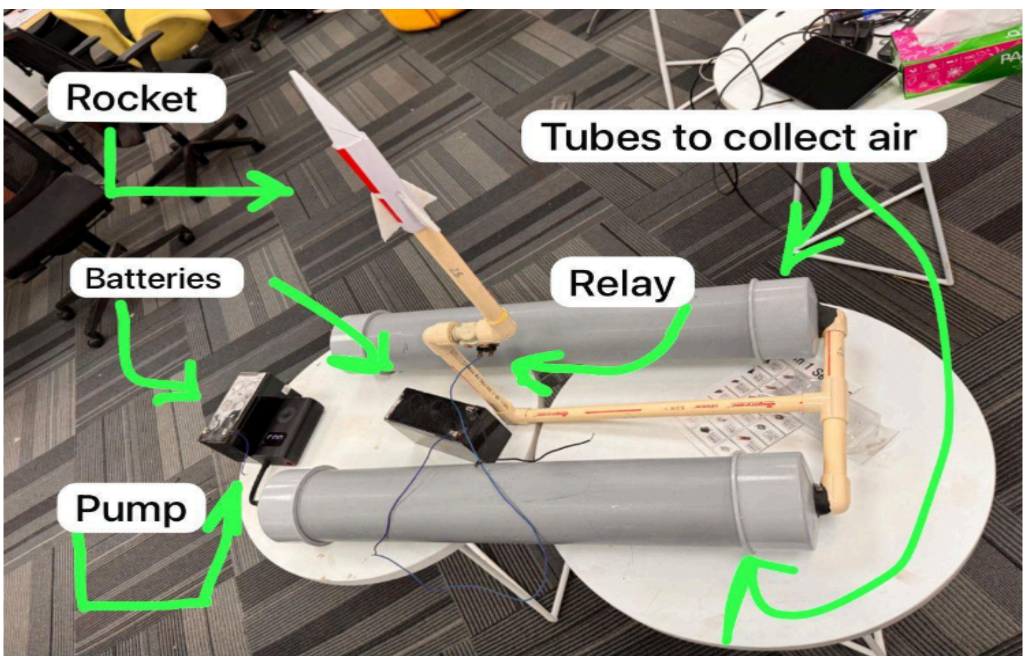


Figure No 4: The launchpad of air based propulsion.

Rocket body is used from lightweight materials such as cardboard. The pressure pump's output is connected to PVC Pipes based chambers. The launch mechanism is a bicycle pump or air compressor. Super Glue is used for attaching fins and other parts secure. The releasing part is an electronic solenoid valve with a battery and remote switch used for Launch, and the launching pad was a wood based platform.

3.6.2 CO2 based Rocket Propulsion System:

The used CO2 cartridge has a specification of 8.8 cm of length, 2.2 cm of diameter, 0.6 cm of nozzle diameter and 16 gm of mass. Nozzle diameter 0.1 mm. The release mechanism is a metal pin. The launch pad was a wood based platform. The designed CO2 propulsion system is in figure no 5.

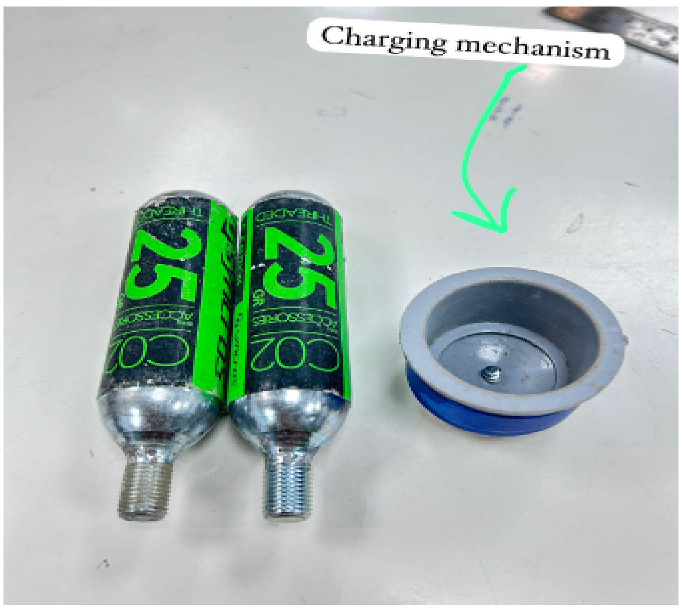
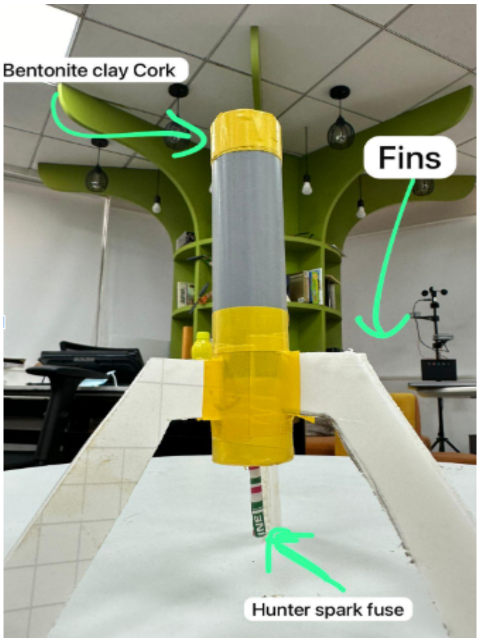


Figure No 5: The propulsion system of CO2

3.6.3 Chemical based Rocket Propulsion System

Sugar is used as a fuel with Potassium Nitrate(KNO_3) as an oxidiser. To start the combustion process, hunter sparkfuse is used as an ignitor.



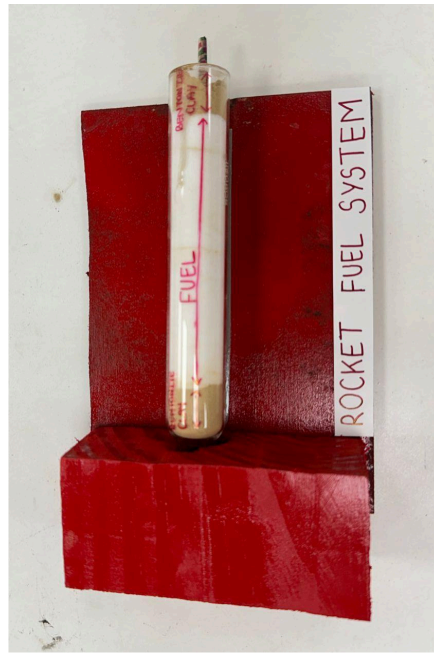


Figure No 6: Chemical Based Propulsion System(left) and Rocket Fuel Container(right)

3.7 Launching the Rocket

In terms of body design, the emphasis on lightweight construction. For air based propulsion, the application of a solenoid as a switch with the help of a 7 V battery was an innovative idea. The solenoid stops the air flow between the presserize tube to the chamber, then the switch is on, allowing the air to flow.

Regarding the rocket launcher, utilizing a CO2 hole puncher, which used a pin and a spring, the spring stays in the compressed state unless a small current passes through the spring, a 7 V battery is used to provide the current. When the current passes through the spring, the mechanism releases the spring which inturn hit the CO2 nozzle and punchures it.for release mechanism showcases creative thinking in repurposing everyday objects for unconventional uses For the chemical based rocket, the application of sugar as a fuel, is an innovative idea, use of day-to-day life material and getting it used as fuel.

4.0 Results

4.1. Theoretical Estimates of the Height attained

In this section, theoretical estimates of the attained height will be calculated for each of the propulsion systems.

4.1.1 Air-based propulsion systems

We will use the concept of impulse provided by the pressurized air-propulsion system to the rocket at initial. The pressure inside the air propulsion system is 3 psi, the radius of the propulsion chamber is 1 cm and the mass of the rocket is 50 gm. The impulse provided to the rocket can be mathematically calculated as:

The pressure provides an upward thrust of $F = P \times A$, where A is the area of the combustion chamber. Therefore,

$$F = P \times A$$

$$p = P \times A \times t$$

where p is the momentum and t is the interaction time.

As the rocket was at rest at initial, $p = mv$, where v is the velocity of the rocket at launch and m is the mass of the rocket.

$$v = (P \times A \times t)/m$$

velocity with which the rocket is launched.

The kinetic energy provided to the rocket is:

$$K.E. = \frac{1}{2} \times m \times v^2 = (PAt)^2/2m$$

(kinetic energy of the rocket at initial)

The maximum height attained by the rocket will be also the turning point for the rocket, for which the velocity becomes zero, at maximum height. Thus, the kinetic energy becomes potential energy at the top, thus,

$$P.E. = K.E.$$

$$mgh = (PAt)^2/2m$$

Therefore,

$$h = (PAt)^2/2gm^2$$

The theoretical height attained by the rocket.

Substitution the values:

P = 3 psi = $3 \times 6895\ Pa$

r = 1 cm

t = 0.1 sec

m = 50 gm

g = 10 m/s^2

$$h = (PAt)^2/2gm^2$$

$$= (3 \times 6895 \times (3.14 \times 10^{-4}) \times 0.1))/(2 \times 10 \times (0.05)^2)$$

$$\text{h} = 8.437 \text{ m}$$

Therefore, the theoretical height attained by the rocket must be 8.437 m

4.1.2 CO2 based propulsion systems

To measure the thrust provided by the CO2 propulsions systems, we will consider a model in which the CO2 is coming out from the cartilage at a constant velocity. The thrust generated by the propulsion system can be measured as: $F = v \times dm/dt$. The rate of mass change can be calculated by using fluid dynamics.

The flow of mass (dm) out of the cartilage in a small interval of time (dt), can be written as,

$$dm = A \times \rho \times (x_2 - x_1) = A \times \rho \times v \times dt$$

$$\text{but, } x_2 - x_1 = v \times dt$$

Where,

A is the area of the nozzle

ρ is the density of the CO2

v is the velocity of ejection of the CO2

$x_2 - x_1$ is the short interval traveled by the liquid in the time interval dt

Thus, dm/dt = $A \times \rho \times v$, implies, $F = v \times dm/dt = A \times \rho \times v^2$

This is the thrust generated by the CO2 propulsion. Thus, the acceleration of the rocket will be:

$$a = (A \times \rho \times v^2)/(m + M)$$

Therefore, the net acceleration will be a - g, vertically upwards,

$$v(t) = [(A \times \rho \times v^2)/(m + M) - g] \times t$$

$$h(t) = \frac{1}{2} \times [(A \times \rho \times v^2)/(m + M) - g] \times t^2$$

The last step is to estimate the average velocity of the CO2, for this we have measured the amount of time it will take for the CO2 cartridge to empty out. It takes about 8 sec for that to happen,

From dm/dt, dm/dt = $A \times \rho \times v$:

$$dm = A \times \rho \times v \times dt$$

Integrating both sides from t = 0 to t = 8 seconds, we get,

$$\int_0^{m=16gm} dm = \int_0^6 A \times \rho \times v \times dt$$

16gm $= A \times \rho \times v \times 6$, implies that $v = 0.016/(A \times \rho \times 6)$.

Substituting,

$$r = 0.07mm = 7 \times 10^{-5} m,$$

$$A = \pi r^2$$

$$\rho = 1100\ kg/m^3$$

We get,

$$v = 0.016/(A \times \rho \times 6)$$

$$= 0.016/(\pi r^2 \times \rho \times 6)$$

$$= 0.016/(3.14 \times (7 \times 10^{-5})^2 \times 1100 \times 6)$$

$$= 157.56 \text{ m/s}$$

using v to calculate a we get,

$$a = (A \times \rho \times v^2)/(m + M)$$

$$= (3.14 \times 10^{-4} \times 1100 \times 157.56^2)/((16 + 16) \times 10^{-3})$$

$$= 13.13 \text{ m/s}^2$$

The rocket's mass has been reduced to 16 gm, for this propulsion.

As the CO2 runs for 8 seconds, the velocity and the distance covered by the rocket in the first 8 seconds of its flight would be.

$$v(t) = [(A \times \rho \times v^2)/(m + M) - g] \times t$$

$$h(t) = \frac{1}{2} \times [(A \times \rho \times v^2)/(m + M) - g] \times t^2$$

Becomes,

$$v(t) = [13.13 - 9.8] \times 6 = 19.8$$

$$h(t) = \frac{1}{2} \times [13.13 - 9.8] \times 6^2 = 59.4 \text{ m}$$

Thus, once the empty out the distance covered by the rocket would be,

$$0^2 = v^2 - 2sg$$

Imply that, $s = v^2/2g = 19.8^2/2 \times 9.8 = 20\,m$

Therefore the estimated height is 59.4 m + 20 m, or about 80 m

4.1.3 Chemical Based propulsion

The chemical used for chemical based propulsion sugar is used as a fuel and potassium nitrate is used as an oxidiser. For the launch 25 gm of sugar is used with 75 gm of potassium nitrate. When performed in the lab, the provided thrust is 3 N, upwards. The mass of the rocket is 200 mg, imply the upward acceleration is $a = 3/0.2 - 9.8 = 5.2\,m/s^2$, and the fuel ran for 6 seconds. The height attained can be calculated as:

$$v = a \times t = 5.2 \times 6 = 31.2\,m/s$$

$$h = \frac{1}{2} \times a \times t^2 = \frac{1}{2} \times 5.2 \times 6^2 = 93.6\,m$$

Thus after 6 seconds, the attained height is:

$$s = v^2/2g = 31.2^2/2 \times 9.8 = 49.6\,m$$

Thus, the estimated attained height is $h = 93.6 + 49.6 = 143.3$ m

Thus the theoretical height attained by the rocket with the chemical propulsion is 143.3 m.

4.2 Observation

For each propulsion system, a total of five trials are performed, i.e. the rocket is launched five times per propulsion. The observed and estimated height is provided in below tables.

4.2.1 Air-based propulsion

Sr No	Trials	Height Estimated (in m)	Height Observed (in m)	Efficiency
1	First	8.44	5.0	0.59
2	Second	8.44	3.0	0.35
3	Third	8.44	6.0	0.71
4	Fourth	8.44	5.5	0.65
5	Fifth	8.44	4.5	0.53

Table No 1: Efficiency of air-based propulsion system

4.2.2. CO2 based propulsion

Sr No	Trials	Height Estimated (in m)	Height Observed (in m)	Efficiency
1	First	80	15	0.18
2	Second	80	18	0.22
3	Third	80	25	0.31

4	Fourth	80	30	0.37
5	Fifth	80	35	0.43

Table No 2: Efficiency of CO2 based propulsion system

4.3.2. Chemical based propulsion

Sr No	Trials	Height Estimated (in m)	Height Observed (in m)	Efficiency
1	First	143.3	30	0.20
2	Second	143.3	50	0.34
3	Third	143.3	40	0.27
4	Fourth	143.3	40	0.27
5	Fifth	143.3	60	0.41

Table No 3: Efficiency of Chemical based propulsion

5.0 Calculations

5.1 Air-Based

The average of the efficiency can be calculated as:

$E_{average} = (0.59 + 0.35 + 0.71 + 0.65 + 0.53)/5 = 0.56$

(average efficiency)

The standard deviation of efficiency can be calculated as:

$E_{deviation} =$

(standard deviation)

$\sqrt{[(0.59 - 0.56)^2 + (0.35 - 0.56)^2 + (0.71 - 0.56)^2 + (0.65 - 0.56)^2 + (0.53 - 0.56)^2]/5}$

$= 0.12$

Therefore the average and standard deviation of efficiency are 0.56 and 0.12, respectively.

5.2 CO2 based

The average of the efficiency can be calculated as:

$E_{average} = (0.18 + 0.22 + 0.31 + 0.37 + 0.43)/5 = 0.30$

(average efficiency)

The standard deviation of efficiency can be calculated as:

$E_{deviation}$ =
(standard deviation)

$$\sqrt{[(0.18 - 0.30)^2 + (0.22 - 0.30)^2 + (0.31 - 0.30)^2 + (0.37 - 0.30)^2 + (0.43 - 0.30]/5}$$

= 0.092

5.3 Chemical based

The average of the efficiency can be calculated as:

$E_{average}$ = $(0.20 + 0.34 + 0.27 + 0.27 + 0.41)$/5 = 0.30

(average efficiency)

$E_{deviation}$ =
(standard deviation)

$$\sqrt{[(0.20 - 0.30) + (0.34 - 0.30)^2 + (0.27 - 0.30)^2 + (0.27 - 0.30)^2 + (0.41 - 0.30)]/5}$$

= 0.07

6.0 Result

The efficiency calculated for each of the different propulsion systems are calculated and provided below.

(i) Air-based propulsion system - the efficiency is 0.56 +/- 0.12 .

(ii) CO_2-based propulsion system - the efficiency is 0.30 +/- 0.09.

(iii) Chemical-based propulsion system - the efficiency is 0.30 +/- 0.07.

7.0 Conclusion

(i) Rocket propulsion system is modeled.

(ii) A launching system is created for every propulsion system.

(iii) The launches of rockets with different propulsion systems are studied.

(iv) The efficiency is calculated for each propulsion system.

(a) air-based propulsion system: the estimated efficiency is 0.56 +/- 0.12. The loss of energy can be due to the fact that the impulse providing air-pressure, also contributes to some horizontal impulse, which does not lead to generating thrust in the rocket.

(b) CO_2-based propulsion system: the estimated efficiency is 0.30 +/- 0.9. The loss of energy can be due to the fact that due to such a small size of the nozzle, a component of the force would be along the horizontal direction. The assumed model, which is a constant velocity mode, may need slight modifications, such as a variable velocity.

(c) Chemical-based propulsion system: the estimated efficiency is 0.30 +/- 0.07. The loss of energy can be because the provided impulse may have a horizontal component.

(v) This experiment can lay a simple modeling of efficiency of different propulsion systems, based on this foundation, much bigger propulsion systems can be made.

8.0 Limitations

(i) The measurements are done in open ground, no control over air resistance and wind's speed.
(ii) The shape of the rocket will also change the efficiency which can be related to stability of the rocket. A simple shape of the rocket is considered.
(iii) The burning rate of chemical based propulsion also depends on the packing method and moisture present in the air. We have tried to keep the moisture low by doing the experiment near noon time, and the packing method is the same for all the trials.

9.0 References

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