Model Rocketry and How it Coincides with Rocket/Aerospace Engineering at Large

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Abstract

Model rocketry is not only an enjoyable activity for many but also represents a valuable teaching tool for new rocketeers. Although model rockets are significantly smaller than rockets sent to outer space, one can still observe the crucial math needed to construct and fly these complex vehicles. The underlying math is exemplified through the ideal rocket equation. This equation helps to demonstrate many aspects of a rocket's flight and can strongly aid in choosing engines and other structural components. This paper will delve into the process of utilizing this equation for amateurs who may not be familiar with the mathematical side of hobby rocketry and develop a true experiment with the application of this math. This experiment equips the Estes Mix-N-Match 55 model rocket to demonstrate the advantages of a multistage rocket and the importance of the rocket's engine. Ideally, experiments similar to the one discussed in this paper will inspire future generations of rocket engineers to continue this work throughout larger situations. Model rocketry is an exciting way to interest both adolescents and adults alike in the complexities of rocket engineering.

Historical Background

Rocketry has become a major part of modern technology without many realizing it. It's typically thought of in the context of space exploration, but rockets are also used for fireworks, missiles, ejection seats, and other human creations. But how and when did this technology become so well-known? Although humans have been studying astronomy for thousands of years, the concept of the rocket is relatively new, especially the type that engineers have come to study today.

In early warfare (c. 1400), nations would fuel rockets with gunpowder and cause projectiles to explode over enemy lines. One would pack a bamboo stick full of gunpowder and attach a long spear to be launched when the gunpowder was ignited. However, these rockets were largely inaccurate because the math behind them wasn't understood or developed yet. Rockets still appeared in battles throughout history from time to time, but alas, interest in rocketry took a pause as new weapons technology, such as cannons and rifles, were developed [1].

Nevertheless, there was still hope for a future of rocketry that was brought to life through Konstantin Eduardovich Tsiolkovsky. In 1903, Tsiolkovsky published a book entitled "Investigating Space With Rocket Devices" that described how rockets could be used to launch orbital spaceships and developed what is now known as the ideal rocket equation. This equation demonstrates the relationship between a rocket's velocity, the speed of the mass being ejected, and the mass of the rocket. Figure 1 represents a visual depiction of Tsiolkovsky's ideas that demonstrates the expulsion of fuel/mass from inside the rocket to propel a vehicle forward. He also theorized the possibility and benefits of a multistage rocket in the 1920s. Tsiolkovsky never lived to see the application of his math to launch rockets; however, his work lived on to become globally recognized throughout the 1950s and 1960s and became a basis for engineers to work upon [1].

Figure 1: Rocket concept illustration by Tsiolkovsky [2]

Interestingly, what is now known as model/hobby rockets also took a large part in the development of larger-scale rockets and demonstrated much of Tsiolkovsky's theorized math. A strong example of this occurred in Germany during the 1920s and 1930s. In 1923, Hermann Oberth published a book entitled *The Rocket into Interplanetary Space*, which detailed the mathematical theories of rocketry and offered designs for rockets [3]. Although successful, his book was extremely complex, which caused Max Valier to adapt it into a simpler format. This book helped to inspire a wave of rocketry in Germany and many attempted to replicate Oberth's theories in actuality, including the Society for Spaceship Travel. Under Valier's leadership, the society was able to build a rocket-powered race car in 1929 which helped to maintain public attention. However, one of the largest successes came in 1931, when the society launched a model rocket to an altitude of about 2,000 feet (610 meters). One model rocket in particular, the Mirak rocket shown in figure 2, even convinced the German military that rockets could be a valuable asset in war [4]. As one can clearly see, this rocket follows a similar build to amateur rockets built today, therefore demonstrating their relevance throughout history. These rockets inspired deadly weapons and convinced many in Europe, and eventually the United States, how useful rocket technology could become. It also demonstrated math that was only theorized, which gained publicity and displayed rocketry as a subject that had the potential to become extremely widespread and influential.

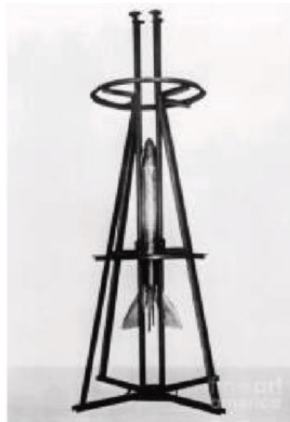

Figure 2: Mirak Rocket [5]

This history has inspired this paper in multiple ways. First, there was clearly a need to break down the math and make it accessible to younger generations. The goal of this project is to allow people without formal education in rocket engineering the ability to also enjoy its successes and challenges. Secondly, history has affected the exploration of model rockets and how those can demonstrate many similarities to rocketry on a larger scale. Many only understand model rockets to be something exciting they can actively enjoy. While this may be the case for many, model rockets also help to provide real-world examples for the math that can typically only be read about. Lastly, the story of spaceflight and rocketry has taken many different turns and follows an interesting road through history. This rocky history has intrigued many and this paper is aimed to continue that fascination.

Analysis

Before discussing the math that can be utilized to analyze rockets, it is important to fully understand what a rocket is. A rocket is any vehicle that propels itself forward by ejecting mass, regardless of the propulsion technology. With an understanding of basic physics, this can be understood through Newton's third law, which states that every action has an equal and opposite reaction. As mass is expelled through the nozzle of a rocket, it is thrust forward to provide the opposite reaction.

$$p = mv \tag{1}$$

Also, following the law of conservation of momentum, if mass (*m*) is ejected/decreased, the velocity (*v*) must increase.

$$(m - dm)(v + dv) + dm(v - u) = mv \tag{2}$$

To begin analyzing a rocket, one must follow the ejection of a small amount of mass (*dm*) over a small period of time (*dt*). The rocket is traveling with an initial velocity of *v* and gains a velocity of *u* following the ejection of mass. When writing an equation for momentum conservation, one must consider the momentum of the rocket and the momentum of the fuel. The rocket loses mass *dm* , meaning its velocity must increase. The total output of fuel is increased to dm which means the velocity is decreased to become a negative value. This value can be found by subtracting the initial velocity of the rocket from the final velocity of the rocket. With this knowledge, Eq. 2 is developed utilizing the law of conservation of momentum.

Then the terms can be expanded to cancel many of them out. Second-order terms should also be eliminated as they are extremely small quantities. The quantity of mass becomes negative because it is being expelled from the vehicle. This leads to Eq. 3

$$m(dv) = -dm(u) \tag{3}$$

Then by dividing *m* over to the right had side of the equation, the definite integral of each side can be produced (Eq. 4):

$$\int_{vi}^{v} dv = -u\int_{mi}^{m} \frac{dm}{m} \tag{4}$$

This can be simplified to become our final rocket equation: Eq. 5

$$v - v_i = u\ln\left(\frac{m_i}{m}\right) \tag{5}$$

Eq. 5 explicitly displays the maximum velocity and makes it easier to find the maximum altitude a rocket can reach with the amount of fuel used [6]. This equation becomes useful in many aspects of rocketry including describing the change in velocity, identifying the ideal mass (and therefore construction), and aiding in engine choice. If used correctly, the ideal rocket equation can be used to significantly improve the design of a rocket. It is also very important to understand the mathematics that describes the influence of mass on a rocket. Initial mass (m_0) can be broken down into three components: structural mass (m_s), payload mass (m_*), and propellant mass (m_p). Similarly, the final mass (m_f) can be broken down into payload mass and structural mass (propellent mass will no longer be a factor since it should all be burned). These values can then be put into the mass ratio (m_f/m_0) to determine how the rocket will be affected by mass quickly. A higher mass ratio means that a rocket can carry more payload and travel further and the opposite is true for a lower mass ratio. There are a few other helpful ratios to note. The payload ratio measures the amount of payload in the initial vehicle ($\pi = m_*/m_0$) [7]. Ideally, the payload ratio would be a large number to result in a greater change in velocity, as there is less unused weight. The structural ratio measures how much of the initial vehicle is structural mass ($\epsilon = m_s/(m_s + m_p)$) [7]. For this ratio, a smaller number would demonstrate higher efficiency because this would demonstrate less unused weight as the structural mass is in the denominator. These equations can also be modified to evaluate multi-stage rockets.

Multistage rockets - although appearing a bit more complex - are the most advantageous forms of rockets, as illustrated throughout historical and current rocket technology. These types of rockets help to discard extra mass, therefore further propelling the rocket and eliminating unused structural mass. The math is similar to that of single-stage rockets; the only difference is that each stage must be looked at separately. Each stage should be treated as if it were a single-stage rocket. To see the overall payload ratio, each of the individual payload ratios would be multiplied by each other:

$$\prod_{k=1}^{N} \pi_k \tag{6}$$

By multiplying the individual payload ratios, an even smaller number can be reached to generate a larger burnout velocity. As a result of this math, it becomes clear that a rocket with multiple stages is preferable to a rocket with only one. However, as the number of stages increases, the burnout velocities increase to a limit. Multiple stages eventually reach an efficiency limit, so rockets are typically built with around three. This can be seen in the graph below:

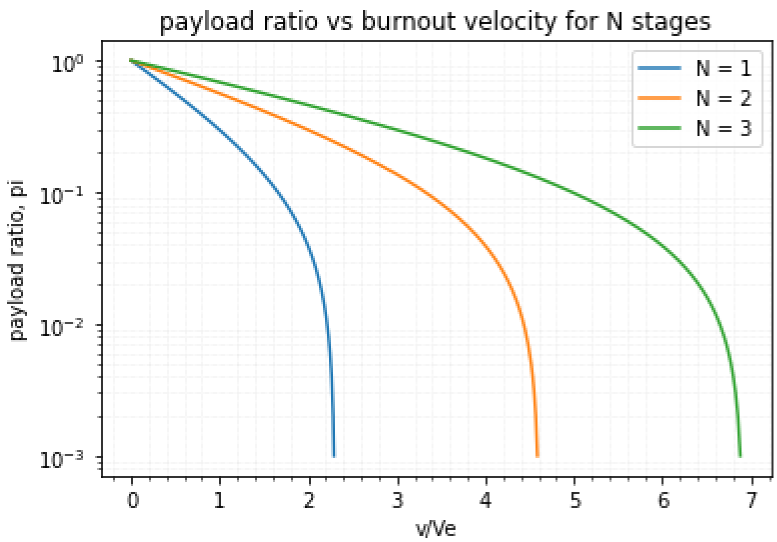

Figure 3: Graph of Payload Ratio vs. Burnout Velocity for *N* number of stages

From figure 3, it is clear that the burnout velocity nearly doubles from one stage to two stages, demonstrating an advantage. Although this graph does not evaluate more stages than three, the graphs do begin to reach a limit as N approaches infinity. It quickly becomes inefficient to build a rocket with more stages because it is not worth the time, complexity, cost, and effort to add stages for a very minimal increase in burnout velocity.

Experimentation

Although multi-stage rockets are superior to single-stage rockets in the mathematical analysis, the experiment below tests two single-stage model rockets that are still beneficial to beginning an engineer's journey into rocketry. This experiment uses a model rocket to test theories that were devised from math. Several areas of thought went into designing this experiment. Firstly, a constant structural component had to be chosen. Estes Mix-N-Match 55 model rocket was chosen because it had a more simplistic build and would allow more time to focus on the technical aspects rather than the construction aspects. With this kit, many younger generations could begin to enjoy rocketry without having to overcomplicate aspects of it. Estes created a (discontinued) kit with three easy-to-build rockets that allowed for multiple experiments and observations. The only supplies that were not included were engines (discussed below), protective wadding, batteries for the igniter, the launch pad, and glue. Initially, Gorilla Glue was utilized, however, it was quickly discovered that it did not dry fast enough nor was it strong enough to hold the rocket together. Once the switch was made to super glue, the rocket become simple to build. By following the instructions provided in the kit, two model rockets were built within one day.

Although the structure and build are extremely important, the rocket would be nothing without its engine. Estes offers a variety of compatible engines to choose from for the rockets including both B and C engines. To decide which engine would be most beneficial, one should utilize the structural ratio. For Estes' rocket engines, the C type has a mass of 12.2 grams, the B type has a mass of 6.5 grams, and the rocket has a structural mass of around 85 grams. For the Mix-N-Match 55 rocket in particular, the two engines used were the B6-4 and C6-5. With this information and the knowledge of how to construct a payload ratio, two separate ratios can be created to determine which engine would be more efficient.

$$\text{C Engine: } \epsilon = 0.085/(0.0122 + 0.085) = 0.874 \quad (7)$$

$$\text{B Engine: } \epsilon = 0.085/(0.0065 + 0.085) = 0.929 \quad (8)$$

Although Eq. 7 and 8 demonstrate that the C engine is more advantageous mathematically, this data can also be represented in a graph. It is important to note that the rocket equation is an exponential relationship which can be viewed clearly on a logarithmic scale. The graph below was created using Python.

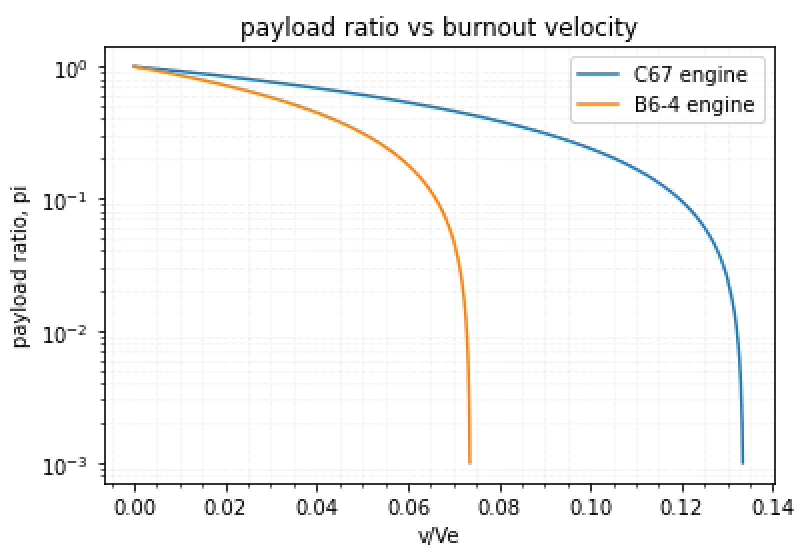

Figure 4: Payload Ratio vs. Burnout Velocity for different Estes engines

It quickly becomes clear (especially when represented in a graph such as figure 4) that a C engine is more efficient than a B engine, but both were tested in actuality to defend this conclusion.

To launch the rockets, one must make sure they are in an area that is open and follows FAA regulations. This experiment was set up far away from any surrounding airports and set up in a large soccer field to ensure no people would be affected by the launch. The launch pad was set in the middle of the field and the first rocket with the B engine was set on the stand. Initially, there was an error with the connectivity of the wires, engines, and electrodes; it was quickly reconfigured and fixed by removing the pin that held the igniters in place and pushing the igniters down a little further to actually touch the engine. With that, the rocket was ready to launch and did not have any further issues.

Figure 5: Rocket launch with B engine at different stages

Figure 5 shows the launch discussed in this paper at different stages. It begins at the launch pad, then is propelled upwards by the expulsion of mass and deploys a parachute after descending from its maximum altitude. The rocket with the B engine reached an altitude of approximately 700 feet and reached an altitude of approximately 1000 feet with a C engine. These numbers should not be taken as givens, as they were based mainly on visual reference. However, in launching the two rockets within a few minutes of each other in the same conditions, it was clear that the rocket with the C engine went much higher.

Conclusion

Even in the most basic experiments that involve minimal rocket construction, the math used in larger rockets is still present and can help to increase the results of a model rocket. By clearly outlining the math, younger generations can become interested in rocketry and understand physics in a way they may not have before. Rocketry may have only recently became widely recognized but has been used by humans for an extended period of time. It has become an important aspect to our society, whether that be for developing weapons, exploring space, or any other area it may find itself applicable. Rocketry at its core demonstrates the conservation of momentum, however, the semi-recently defined math has been manipulated to form more beneficial and specific equations. For basic rocketry, the ideal rocket equation is utilized along with many ratios to demonstrate relationships between different aspects of the rocket. These equations are applicable for any rocket, no matter how large or small.

By depicting these key concepts in ways such as model rocketry for many to enjoy, more people can become interested in fields of engineering and physics. Many also have a misconception of the field of rocketry and often overcomplicate it in their own minds. Amateur rocketry can help to break down these barriers by demonstrating that anyone can learn the basics behind launching a rocket. These realizations may form a new era of engineers that have the ability to make great impacts in areas like weapons development or space travel. Helping people realize their untapped abilities can be difficult but with the continued education of those with a passion to learn can usher in a brighter future and advancements in rocketry with the right guidance and resources.

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