Study of Kepler's Laws and Kepler's Constant in Various Asteroids of the Solar System

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Abstract: Kepler's laws of motion are fundamental planetary motion, planets, asteroids, comets follow these laws with no or minimal deviation. In this paper, we perform statistical analysis on Kepler's constant, by using 5350 minor planets data from the minor planet center (MPC). According to Kepler's third law of motion, the square of orbital time period of a minor planet is proportional to cubed of the semi-major axis distance, the proportionality constant is Kepler's constant, $G(M + m)/4\pi^2 = r^3/T^2$, M and m are the mass of the sun and minor planet respectively, and G is the universal constant of gravity. Thus, Kepler constant is also related to the mass of the minor planet, suggesting that Kepler constant depends on 2-body interaction. Our analysis suggests that $Log\,T = 1.5 \times Log\,a + 5.7 \times 10^{-11}$, which is as estimated, the second term of the equation corresponds to the mass of the minor planet, having said that the error in the data was too high, and measurement of the mass of the asteroid was not possible. The average value and the standard deviation of the Kepler's constant are 1 and 8.47×10^{-10}, which suggest the spreading of the constant to be low, also, a plot of histogram of the error suggests that the distribution is a normal distribution centered at zero. Thus the conclusion is that all the minor planets follow Kepler's law with not much diversion, leading us to believe that many body forces also lead to abiding by Kepler's laws at astronomical level.

Keywords: Kepler Constant, Minor Planets, 2-body interaction, Error, Normal Distribution

1. Introduction

The Minor Planet Center(MPC) is the official worldwide organization in charge of collecting observational data for minor planets such as asteroids, comets, etc, and keeping a track of their orbital track information. MPC works Under the International Astronomical Union(IAU), it operates at the Smithsonian Astrophysical Observatory. MPC observations include near-Earth objects (NEO), Orbits for Trans Neptunian Objects(TNOs), Centaurs and Scattered Disk Objects(SDOs), etc. MPC majorly used TESS, Pan-STARRS and the VRO-LSST [8] telescopes used for the data collection. Planets, comets and asteroids obey the law of planetary motion discovered by Johannes Kepler, today it's known as Kepler's laws of motion.

Kepler's laws of motion are planetary laws of motion formulated by Johannes Kepler in the 17th Century [1]. Johannes Kepler was working with Tycho Brahe, they were analyzing the collected observational data of planets with the help of solely naked eyes [2]. Johannes analyzed the data and formulated the three laws that describe the motion of planets in the solar system. The formulated laws are as follows: *First Law of Kepler:* All planets in the solar system move in an elliptical orbit with the Sun at one of the foci. *Second Law of Kepler:* Planets sweep out equal areas in equal intervals of time when orbiting the Sun. *Third Law of Kepler:* The squared orbital time period of the planets around the Sun is proportional to the cube of the distance of the semi-major axis.

In the Solar System, the Sun provides the centripetal force to the planet at a distance of 'a' (semi-major axis distance), the centripetal force is due to attraction due to gravity between the planet and Sun of mass m and M. Therefore, centripetal force is proportional to gravitational attraction between the planets and Sun, this can be mathematically written as, $\frac{GMm}{a^2} = ma\omega^2$, where, G is the universal constant of gravity, M is the mass of the Sun, m is the mass of the planet or $GM = a^3\frac{4\pi^2}{T^2}$, where T is the orbital time period of the planet. Could be rearranged for $T^2 = \frac{4\pi^2}{GM}a^3$ . . . (1). The above equation is the mathematical form of Kepler's third law, the constant is $\frac{4\pi^2}{GM}$ [3]. Asteroids are rocky planetary small bodies, also known as 'minor bodies' [4]. Asteroids are irregular in shape and size and can have diameters from a few meters to 300 km [5]. The masses of some of the largest asteroids is estimated to be of the order of $10^{-10} M_{\odot}$ [6]. Asteroids are studied to understand more about the "origin of life", "The Moon's origin", "origin of water on Earth", "reservoirs of valuable resources", "colonization", "potentially hazardous objects" [7].

In this research paper, we are studying the Kepler's constant that emerges in the third law of Kepler. We will be using around five thousand asteroids data from the minor planet center to perform this study.

2. Methodology

This section discusses the methodology used to conduct the research.

2.1 Aim of the Study

To calculate the Kepler constant using the asteroids of Orbits for Trans Neptunian Objects(TNOs), Centaurs and Scattered Disk Objects(SDOs) [8]

2.2 Research Design

Kepler's constant is calculated using the asteroid data provided in the minor planet center. The data is converted into an excel format for easy-to-use. Around 5300 asteroid data has been analyzed in this study. The study included obtaining the Kepler constant for all 5300 asteroids using their orbital time period and semi-major axis distance, plotting a histogram of the same. Also, an error profile is studied, which includes the deviation of the Kepler constant values from an averaged value and a histogram of the same is obtained for visual representation.

2.3 Tools Used

Python[9]

2.4 Data Collection Procedure

The data of asteroids is downloaded from the minor planet center in the .json file, which was then converted into an excel file for a better readability and easy-to-use, which include filtering of the data, plotting of the data using inbuilt functions in the excel.

3. Discussion

In the derivation of Kepler's third law derived in the equation number 1, the assumed model is that one of the masses is predominately larger than the order, and thus, we had neglected the smaller mass effect on the larger mass. By doing this we consider that the bigger mass is steady and the smaller mass is rotating about the bigger mass in an orbit. In reality, both the masses will have gravitational effects of attraction on each other and the larger and the smaller mass object will rotate about the center of their masses. Technically, this is a two body problem in reality, but the derivation done in equation 1 was done by considering the problem as 1 body problem. We will first derive the relation between orbital time period and semi major distance by considering the system as a two-body system, and we will do this in two ways: (i) Transforming the system from a 2 body to 1 body problem by using chirp mass concept, (ii) By considering the system as a 2 body problem.

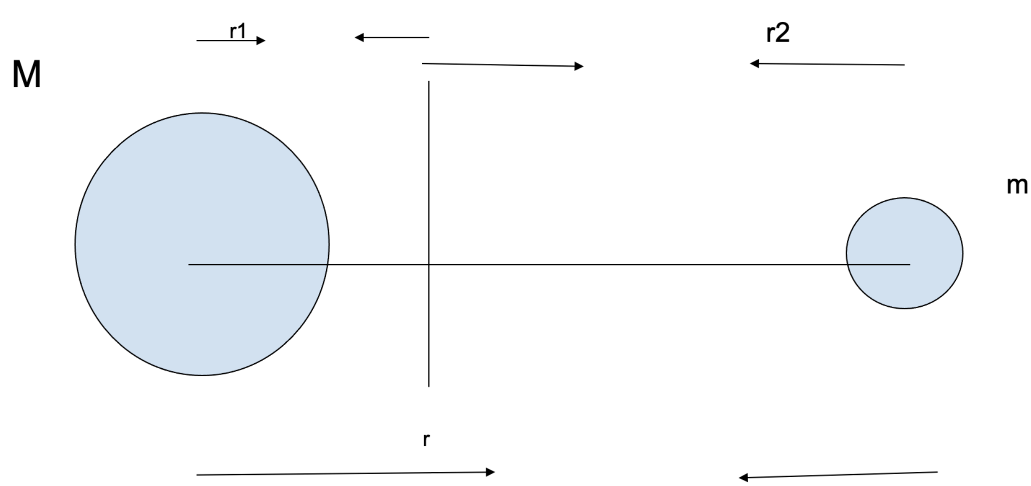

Figure No: 01 - Two objects rotating about their center of mass, which is at a distance of r1 from the mass M and mass m is at a distance of r from the capital mass

(i) Reduce mass - Transforming the system from a 2 body to 1 body problem : A trick method.

Reduce mass for a two body system is defined, $\mu = Mm/(M + m)$, where μ is the reduced mass of the system. The problem shown in figure 1, can be considered as a 1 body problem by assuming the bigger mass to be M and the smaller mass to be equal to the chirp mass of the system, and the smaller mass is then can be considered as that if it's rotating around the bigger mass and the bigger mass is steady.

$$F = \mu r w^2 = GMm/r^2$$

$$(Mm/(M + m)).r.w^2 = (G.M.m)/r^2$$

In this, F is the gravitational force between the two bodies, μ is the reduced mass of the system, r is the distance between the two mass and w is the angular velocity of the system.

This gives us, $r^3.w^2 = r^3.\frac{4\pi^2}{T^2} = G.(M + m)$, $\frac{r^3}{T^2} = G(M + m)/4\pi^2$ . . . (2)

We see that equation 2 also depends on the small-mass object 'm', unlike equation (1), which is independent of the mass of the small-mass object.

(ii) The system as 2 body problem

The force on each of the masses is $F = GMm/r^2$, using Newton's law of gravitation.

The radius of rotation of the mass M is r1, the centripetal force on M is $M.r1.w^2$.

The radius of rotation of the mass m is r2, the centripetal force on m is $m.r2.w^2$

The distance between the two masses is r = r1 +r2.

Thus, for mass M, $F = GMm/r^2 = M.r1.w^2$, gives us $Gm/r^2 = r1.w^2$ . . . (3) ,

Similarly for mass m , $F = GMm/r^2 = m.r2.w^2$, gives us $GM/r^2 = r2.w^2$ . . . (4)

Adding equation 3 and 4, we get,

$$G(M + m)/r^2 = (r1 + r2).w^2 = rw^2 \ldots \text{as r1 +r2 = r}$$

$$G(M + m) = r^3w^2, \text{ gives us, } G(M + m)/4\pi^2 = r^3/T^2 \ldots (5)$$

Equations 2 and 5 give Kepler's third law of motion for M and m rotating about their center of mass. In this study we will use equation (5), to calculate the mass of the asteroids (m), by taking the mass of Sun to be $1.98\times10^{30}Kg$.

4. Result

1.Histogram of Absolute Error in Kepler Constant

Absolute Error histogram provides a distribution of error. It is produced by calculating the average of T^2/a^3 for each of the asteroids and finding an average of it. The absolute error in Kepler constant was then defined as Absolute Error = $(T^2/a^2)_{average} - T^2/a^3$. There are a total of about 530 datasets, that is asteroids data used for the histogram plotting.

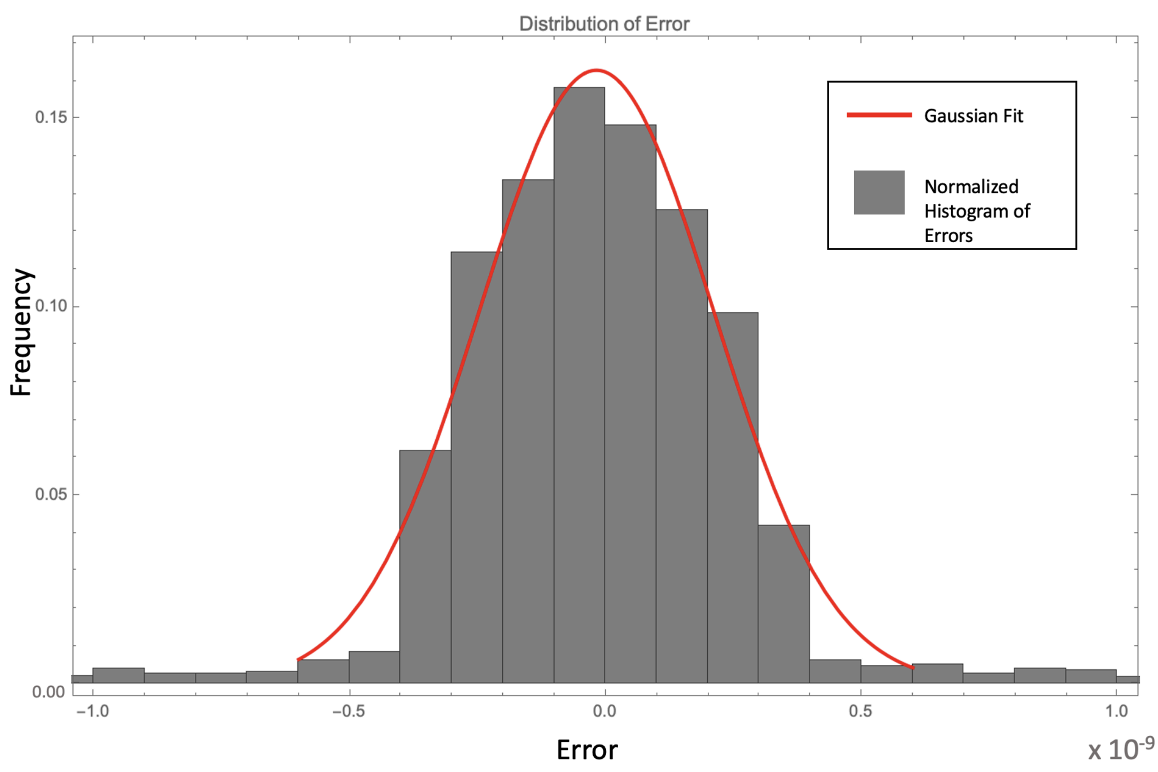

Figure 2: Histogram of Absolute Error

2. Plotting of Log T vs Log A

According to Kepler's third law, with neglecting the mass of the planet, which suggest that $T^2 = \frac{4\pi^2}{GM}a^3$, if we take T = 1 year, a = 1 AU, M = 1 Solar mass, then $4\pi^2/G = 1$, thus, in this customized units of system, $T^2 = a^3$. Taking Log on boths sides we get, $Log(T^2) = Log(a^3)$, can be simplified as , $\frac{LogT}{Loga} = \frac{3}{2}$, thus for a Log T vs Log A plot, the slope should be 3/2 or 1.5.

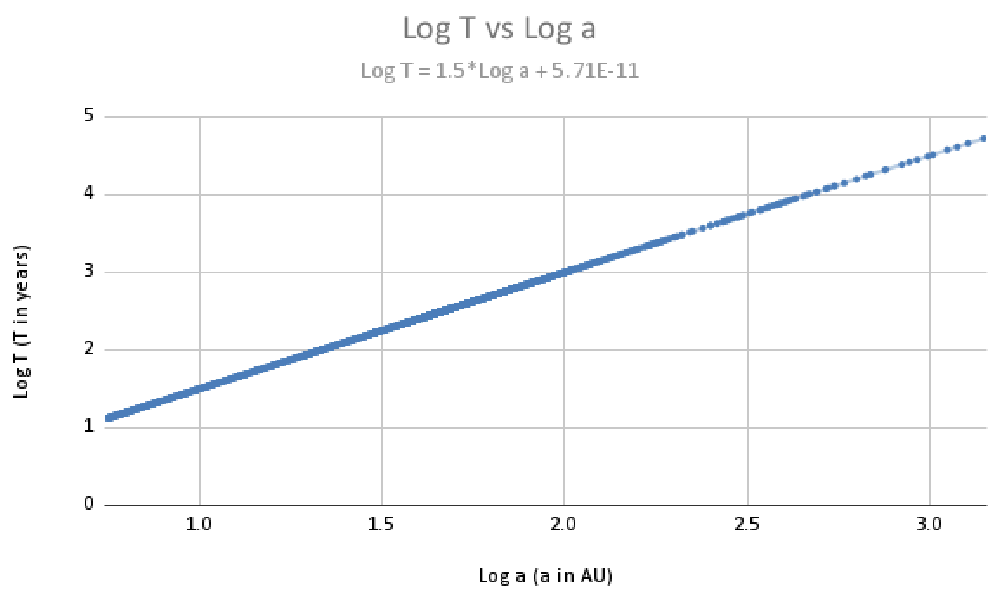

Figure 3: Log T vs Log A scatter plot fitted with a linear trendline.

A scatter plot of the Log T vs Log a is produced, a linear trendline fits the curve, the equation of the curve is $LogT = 1.5\times Loga + 5.71\times10^{-11}$. Thus the slope is 1.5, with an extra term 5.71×10^{-11}which can be attributed to the presence of the planet.

3. Average and Standard Deviation

We can see that the majority of the planets have absolute error of Kepler's constant present quite in a range of [-0.9×10^{-9}, 0.9×10^{-9}], and thus we can say that the distribution is almost centered at about zero. With average, and standard deviations of the absolute error are -5.05×10^{-13} and 8.47×10^{-10}. We have used the orbital data of TNO's and SDO's gathered from MPC. The orbital data includes parameters such as eccentricity, inclination of the orbital plane, orbital period, aphelion and perihelion distances, etc. For data analysis, we have used orbital time period and semi-major axis distance. The masses of the planets can be calculated by using the Kepler's third law, $G(M + m)/4\pi^2 = r^3/T^2$, where M is the mass of the Sun and m is the mass of the planet. The data has a high value of errors, due to which the mass of the planet could not be estimated with accuracy. Nevertheless, it can bee seen from the figure no 3, that the Log T vs Log a, graph have can be fitted with a linear equation of $Log\,T = 1.5\times Log\,a + 5.7\times10^{-11}$ The extra term, which is 5.7×10^{-11}, is due to the presence of the planet.

The average value and the standard deviation of Kepler's constant are 1 and 8.47×10^{-10}. Thus, the obtained Kepler's constant of the planets have a very close distribution; as the standard deviation is small. The distribution of the absolute error in Kepler's constant is provided in figure no 2. It can be seen that the distribution of the absolute error is centered about zero and spreads to zero quickly as one moves away from zero, the average absolute error is -5.05×10^{-13} and the standard deviation of absolute error is8.47×10^{-10}.

5. Conclusion

The statistical analysis of error distribution was studied for Kepler constant. The error follows normal distribution, hinting that constant has a gaussian random characteristic, which hints in the masses of asteroids being Gaussian distributed. Lastly, the minor planets, however small or big, follow Kepler's law with no deviation, despite many body interactions and different gravitational pulls due to varied masses.

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7. References

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