Satellite Rendezvous Scheme

Raaid Ahmed Baloch

A rendezvous in the context of this paper is a set of orbital maneuvers applied to a spacecraft which arrives at a very close distance to the orbit of another satellite. This paper explores a method to help simplify deriving the rendezvous scheme for satellites. Potential applications of this project include utilizing satellite rendezvous design for service missions, space supply and space debris cleaning.

Introduction:

Satellite rendezvous is crucial for various space missions but presents challenges due to complex orbital dynamics. This paper proposes a novel scheme that combines the strengths of Clohessy Wiltshire (CW) equations and Hohmann transfers for more efficient and precise rendezvous.

Traditional methods, like CW equations and Hohmann transfers, are used to independently solve long range and close range problems. This approach aims to integrate both methodologies, enhancing efficiency and accuracy while minimizing fuel consumption. Starting with a detailed analysis of CW equations, then introduction of Hohmann transfers, and adapting them for rendezvous scenarios. This scheme focuses on optimal trajectory design, mission flexibility, and performance evaluation through simulations and numerical analyses. By bridging theory and practice, this research aims to advance satellite rendezvous techniques, offering insights for more efficient and reliable space missions.

The Clohessy-Wiltshire equations, also known as the Clohessy-Wiltshire-Hill equations or simply the Hill-Clohessy-Wiltshire equations, are a set of linearized differential equations that describe the relative motion of 2 spacecraft in a close-proximity, nearly circular orbit around a central body, such as Earth. The equations are commonly used in orbital mechanics, particularly for analyzing the dynamics of spacecraft in formation flying or rendezvous and docking scenarios. The Clohessy-Wiltshire equations are derived by linearizing the equations of motion surrounding the relative motion of two objects in orbit around a central body. They are named after Robert H. Clohessy and Richard S. Wiltshire, who first presented them in their 1960 paper titled "Terminal Guidance System for Satellite Rendezvous” [7]. The Clohessy-Wiltshire equations are used in scenarios where the relative distances and velocities between two spacecraft are small compared to the orbital radius and velocity of the central body. In such cases, the gravitational effects of the central body dominate the dynamics, and the relative motion can be accurately approximated by these linearized equations. The Clohessy-Wiltshire equations are represented in matrix form, which facilitates numerical integration and analysis.

The matrices involved in the equations represent the linearized dynamics of the relative motion and are often referred to as the "Hill-Clohessy-Wiltshire matrices." Overall, the Clohessy-Wiltshire equations and matrices are fundamental tools in orbital mechanics, particularly for analyzing the relative motion of spacecraft in close-proximity orbits and performing tasks such as rendezvous, docking, and formation flying.

The Hohmann transfer is a type of orbital transfer maneuver used to transfer a spacecraft from one circular orbit to another. It is named after the American aerospace engineer John V. Hohmann, who developed the method in the 1950s. The Hohmann transfer is a type of bi-elliptic transfer, meaning it involves two elliptical orbits. The Hohmann transfer is characterized by two main phases: 1) First Burn: In this phase, the spacecraft fires its propulsion system to increase its velocity and transfer to an elliptical orbit with a higher apogee (the point farthest from Earth. 2) Second Burn: Once the spacecraft reaches the apogee of the first elliptical orbit, it performs a second burn to raise its perigee (the point nearest to Earth) to the desired altitude of the target circular orbit. This second burn is usually performed at the apogee of the first orbit to maximize efficiency [2]. The advantage of the Hohmann transfer is that it can be more fuel-efficient than other transfer maneuvers, particularly when the difference in altitude between the initial and target orbits is significant. By utilizing the principles of orbital mechanics, the Hohmann transfer minimizes the overall delta-v (change in velocity) required for the transfer compared to other methods. However, it is important to note that while the Hohmann transfer can be fuel-efficient in certain scenarios, it may not always be the optimal choice depending on mission requirements, such as time constraints or specific orbital characteristics [8] . Overall, the Hohmann transfer is a valuable tool in orbital mechanics for efficiently transferring spacecraft between circular orbits using minimal propulsion resources.

Finally, this paragraph defines all the used orbital elements that have been used in the methods section. The Semi Major Axis (a) defines the size of an orbit, with an increase in the semi-major axis corresponding to an increase in the orbital period [6] . Eccentricity (e) determines the shape of an orbit, ranging from 0 to 1, where 0 represents a circular orbit and 1 represents a parabolic orbit [3]. It illustrates how far or close the orbit can shift from the center of the Earth, with the perigee marking the point where an object is closest to Earth in an elliptical orbit, while the apogee represents the farthest point. Inclination (i) defines the orientation/angle of an orbit on the x-axis (See Figure 1), ranging from 0 to 90 degrees, indicating the degree of tilt [4]. The Right Ascension of the Ascending Node, or the RAAN (Ω) specifies the angle between the reference axis [5] (See the citation or Figure 1) and the satellite’s ascending node. True Anomaly (V) is an angular parameter that delineates the position of a body moving along a Keplerian orbit, representing the angle between the direction of periapsis and the current position of the body as observed from the focus of the ellipse. Additionally the diagram below can be used to better understand how the orbital elements would look with respect to a central figure.

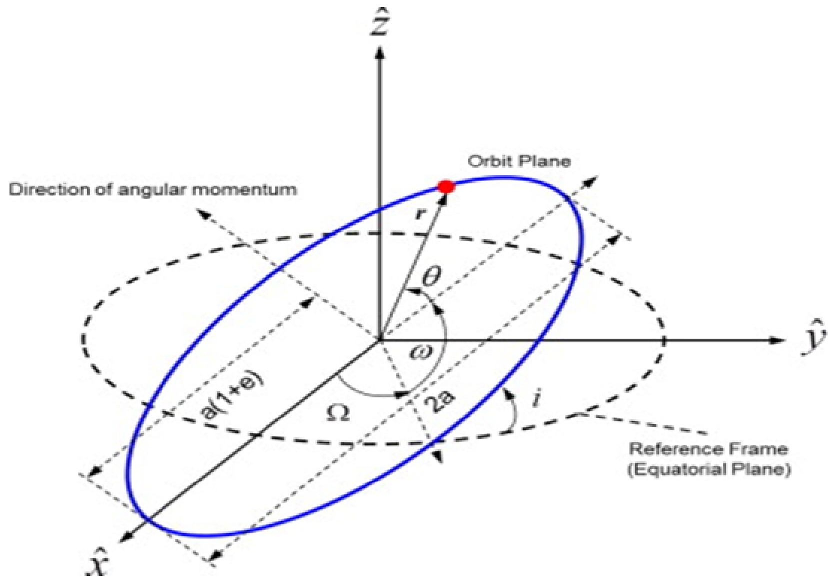

Figure 1: Representation of Keplerian Elements [11]

This paper applies the Clohessy-Wiltshire equations to achieve close range rendezvous after a Hohmann transfer has been performed.

Methods

Derivation of the Hohmann Transfer

Before using examples of a Hohmann transfer in other scenarios it is important to derive the equations for the Hohmann transfer [9]. Starting with the definition of the base value we can take the initial circular orbit radius as r_i, the initial velocity of the spacecraft as v_i and the gravitational constant that is set is 398,000 since earth is taken as the central body.

$$r_a = r_i + \Delta r \tag{2}$$

Starting with equation 2 the parameters for the elliptical transfer orbit are defined. In the equation 2 the apoapsis radius is represented by r_a and is obtained by adding the initial circular orbit radius with the change in radius due to the transfer which is shown by Δr.

$$r_p = r_i$$

(3)

The periapsis radius is set to be equal to the initial radius since the first burn raises the spacecraft into a higher elliptical orbit and the radius to the periapsis has not changed, only the apoapsis.

$$a = \frac{r_p + r_a}{2}$$

(4)

Equation 4 gives the semi major axis which is represented by a and is obtained by taking the average of the periapsis radius and the apoapsis radius.

$$v^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right)$$

(5)

The above equation is known as the Vis-Viva equation and it is used to relate the orbital parameters and the velocities and will be used to calculate the changes in velocity later on.

$$v_c = \sqrt{\frac{\mu}{r}}$$

(6)

Equation 6 calculates the velocity of a spacecraft that is moving in a circular orbit and is represented by v_c. Using this equation and substituting v_c for v_t which we will set as the velocity in the transfer orbit can now be calculated.

$$\Delta V1 = v_t - v_i$$

(7)

Using the equation in step 6 the above equation can be derived. During the first burn the spacecraft accelerates in order to raise the apoapsis. $\Delta V1$ is the needed velocity change which is required for the burn to be performed.

$$v_t = \sqrt{\frac{\mu}{r_i + \Delta r}} \tag{8}$$

Using the equations listed in steps 2 and 6 the value “r” can be substituted in step 6 for the equation listed in step 2 in order to find out the value of v_t.

$$\Delta V2 = v_f - v_t \tag{9}$$

The second burn in a Hohmann transfer is performed at the apoapsis of the transfer orbit and equation 9 shows the change in velocity required for this burn and is represented by $\Delta V2$ and the symbol v_f shows the final circular velocity. Additionally, much like in step 8 the equations can be substituted in step 2 and step 6 in order to calculate v_f .

$$\Delta_{v-total} = \Delta V1 + \Delta V2 \tag{10}$$

Now this final equation will give the total velocity change for the Hohmann transfer which is shown in the symbol $\Delta_{v-total}$. These steps highlight the derivations of the necessary equations used in planning a Hohmann transfer.

Additionally, the figure below plots the trajectory of the spacecraft during a Hohmann Transfer.

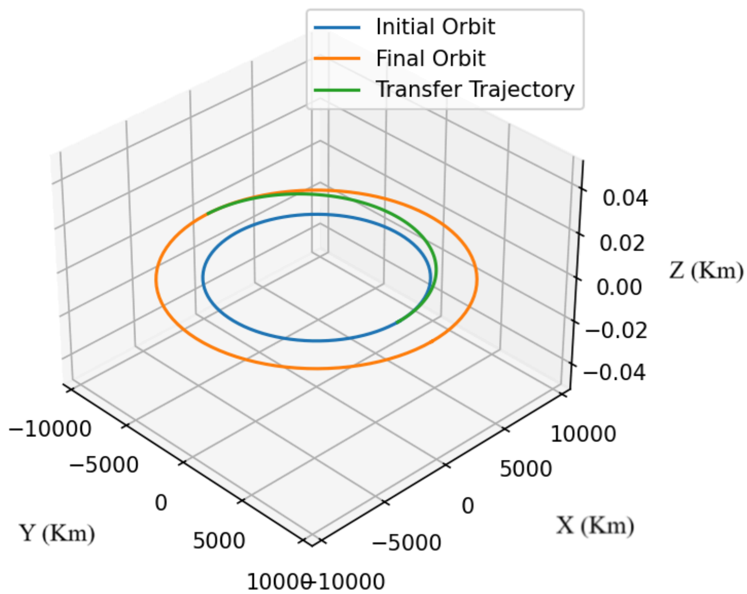

Figure 2: A 3-D representation of a Hohmann transfer

Derivation of Clohessy-Wiltshire Equations:

Once in the vicinity of the second spacecraft, rendezvous must be performed for which the Clohessy-Wiltshire equations need to be applied. This section will derive the Clohessy-Wiltshire equations using a spacecraft and a chaser. [10]

Assumptions to be taken:

The primary body's (The Earth in this scenario) gravitational field is the main force acting on the spacecraft.
The spacecraft is in a circular orbit

Calculations:

$$\frac{d^2 r_t}{dt^2} = -\mu \frac{r_t}{r_t^3} \tag{11}$$

$$\frac{d^2 r_c}{dt^2} = -\mu \frac{r_c}{r_c^3} + f$$

(12)

The above equations are used to show the motion of the target and chaser where r_t shows the position vector of the target and r_c shows the position vector of the chaser. The f however denotes the acceleration caused by the force acting on the chaser.

$$r = r_c - r_t$$

(13)

$$\frac{d^2 r}{dt^2} = -\mu\frac{r_c}{r_c^3} + \mu\frac{r_t}{r_t^3} + f$$

(14)

$$r_c^2 = (r_t + r)^2$$

(15)

Expanding out the brackets in equation 15 and then factoring out r_t^2 will result in equation 16 as shown above.

$$r_c^2 = r_t^2 \times \left[1 + \frac{2r_t \times r}{r_t^2} + \left(\frac{r}{r_t}\right)^2\right]$$

(16)

$$r_c^{-3} = r_t^{-3} \times \left[1 + \frac{2r_t \times r}{r_t^2} + (\frac{r}{r_t})^2\right]^{-\frac{3}{2}}$$

(17)

After raising both sides of the equation to the power $-\frac{3}{2}$ results in equation 17

$$r_c^{-3} = r_t^{-3} \times \left[1 + \frac{2r_t \times r}{r_t^2}\right]^{-\frac{3}{2}}$$

(18)

After taking into account that $r < r_t$ Step 18 is the following result.

$$(1 + x)^p = 1 + px$$

(19)

$$r_c^{-3} = r_t^{-3} \times \left(1 - 3\left[\frac{r_t \times r}{r_t^2}\right]\right)$$

(20)

Using the approximate formula shown in equation 19 we can arrive at the conclusion that is shown in equation 20

$$\frac{d^2r}{dt^2} = -\mu\left(\frac{1}{r_t^3}\right)\left(-3r_t\left(\frac{r_t \times r}{r_t^2}\right) + r\right) + f$$

(21)

Now that a value for r_c^{-3} has been attained step 20's equation can be substituted into the equation of motion displayed in equation 14.

$$\frac{d^2r}{dt^2} = a' + 2\omega\left(v'\right) + \omega\left(\omega \times r'\right) + \dot{\omega} \times r'$$

(22)

When taking into account the position vector in the rotating frame from the time derivative the resulting equation is shown in equation 22.

$$\begin{bmatrix} 2\omega^2 x \\ -\omega^2 \\ -\omega^2 \end{bmatrix} + \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \ddot{z} \end{bmatrix} + 2\begin{bmatrix} -\omega\dot{y} \\ \omega\dot{x} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \omega^2 \end{bmatrix} + \begin{bmatrix} -\omega^2 x \\ -\omega^2 y \\ -\omega^2 z \end{bmatrix}$$

(23)

$$\ddot{x} - 2\omega\dot{y} - 3\omega^2 = f_x$$

(24)

$$\ddot{y} + 2\omega\dot{x} = f_y$$

(25)

$$z + \omega^2 z = f_z$$

(26)

If $\omega = \sqrt{\frac{\mu}{r_t^3}}$ we obtain the matrix equation shown in step 23 , however simplifying that equation will result in the following 3 equations shown in equation 24, 25 and 26 which are the Clohessy Wiltshire equations

Derivation of Clohessy-Wiltshire Matrices

In this portion of the Clohessy Wiltshire derivations the equations that have been derived in the previous section are used to derive the Clohessy Wiltshire matrices which are essential in problem solving.

$$\ddot{x} - 2\omega\dot{y} - 3\omega^2 = 0 \tag{27}$$

$$\ddot{y} + 2\omega\dot{x} = 0 \tag{28}$$

$$\ddot{z} + \omega^2 z = 0 \tag{29}$$

When considering that no force is acting on the chaser initially hence f_x, f_y and f_z in the Clohessy Wiltshire equations can all be considered as equal to 0. Since x and y are coupling these equations can be solved using the elimination method.

$$\dot{y} + 2\omega x = C_1 \tag{30}$$

Differentiating equation 28 will result in obtaining equation 30

$$\ddot{x} + \omega^2 x = 2\omega C_1 \tag{31}$$

The result of substituting equation 30 into equation 27 is shown by equation 31 and now a solvable equation can now be obtained.

$$x = \frac{2C_1}{\omega} + C_2 cos(\omega t) + C_3 sin(\omega t) \tag{32}$$

By rearranging equation 31 to make x the subject equation 32 has been derived

$$\dot{x} = -C_2\omega \sin(\omega t) + C_3\omega \cos(\omega t) \tag{33}$$

Differentiating equation 32 will result in equation 33

$$\dot{y} = -3C_1 - 2C_2\omega \cos(\omega t) - 2C_3\omega \sin(\omega t) \tag{34}$$

Equation 34 is the result of substituting equation 32 into equation 30.

$$y = -3C_1 t - 2C_2 \sin(\omega t) + 2C_3 \cos(\omega t) + C_4 \tag{35}$$

Equation 35 is the result of integrating equation 34 and gives a value for y

$$x_0 = \frac{2C_1}{\omega} + C_2 \tag{36}$$

$$\dot{x}_0 = C_3\omega \tag{37}$$

$$\dot{y}_0 = -3C_1 - 2C_2\omega \tag{38}$$

$$y_0 = 2C_3 + C_4 \tag{39}$$

Equations 36 through 39 is a result of substituting $t = 0$ into equations 32 to 35

$$C_1 = 2\omega x_0 + \dot{y}_0 \tag{40}$$

$$C_2 = -3x_0 - \frac{2\dot{y}_0}{\omega} \tag{41}$$

$$C_3 = \frac{\dot{x}_0}{\omega} \tag{42}$$

$$C_4 = \frac{-2\dot{x}_0}{\omega} + y_0$$

(43)

By rearranging equation 36 to 39 and making C_1,C_2,C_3 and C_4 the subject of the equations we obtain the equations 40 to 43

$$z = C_5 \cos(\omega t) + C_6 \sin(\omega t)$$ (44)

$$\dot{z} = C_5 \omega \sin(\omega t) + C_6 \omega \cos(\omega t)$$ (45)

Since the Z-axis is independent equation 29 must be solved separately

$$C_5 = z_0$$ (46)

$$C_6 = \frac{\dot{z}_0}{\omega}$$ (47)

Equations 46 and 47 is a result of substituting $t = 0$ into equations 44 and 45

$$x = (4 - 3\cos(\omega t))\, x_0 + \frac{(\dot{x}_0)\sin(\omega t)}{\omega} + \frac{(2y_0)(1 - \cos(\omega t))}{\omega}$$ (48)

$$y = 6x_0 (\sin \omega t - \omega t) + y_0 + \frac{(2\dot{x}_0)(\cos \omega t - 1)}{\omega} + \frac{(\dot{y}_0)(4\sin \omega t - 3\omega t)}{\omega}$$ (49)

$$z = z_0(\cos \omega t) + \frac{(\dot{z}_0)(\sin \omega t)}{\omega}$$ (50)

$$\dot{x} = x_0 (3\omega \sin \omega t) + (\dot{x}_0)(\cos \omega t) + (\dot{y}_0)(2\sin \omega t)$$ (51)

$$\dot{y} = 6\omega\, x_0 (\cos \omega t - 1) - (2\dot{x}_0)(\sin \omega t) + (\dot{y}_0)(4\cos \omega t - 3)$$ (52)

$$\dot{z} = -\omega z_0(\sin \omega t) + (\dot{z}_0)(\cos \omega t)$$ (53)

From equations 36 to 39 , 44 and 45 we can obtain the following equations (48 to 53) which can be written in matrix format and shown in equation 54

$$\begin{bmatrix} x(t) \\ y(t) \\ z(t) \\ \dot{x}(t) \\ \dot{y}(t) \\ \dot{z}(t) \end{bmatrix} = \begin{bmatrix} 4-3c & 0 & 0 & \frac{s}{\omega} & \frac{2(1-c)}{\omega} & 0 \\ 6(s-\omega t) & 1 & 0 & \frac{-2(1-c)}{\omega} & \frac{(4s-3\omega t)}{\omega} & 0 \\ 0 & 0 & c & 0 & 0 & \frac{s}{\omega} \\ 3\omega s & 0 & 0 & c & 2s & 0 \\ -6\omega(1-c) & 0 & 0 & -2s & -3+4c & 0 \\ 0 & 0 & \omega s & 0 & 0 & c \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ \dot{x}_0 \\ \dot{y}_0 \\ \dot{z}_0 \end{bmatrix}$$

(54)

Application to a Problem

Now that the derivations above have proven both the Hohmann transfer, the Clohessy Wiltshire equations and Clohessy Wiltshire matrices can now be applied to a rendezvous scenario, the next step is to apply these equations to a problem where an unknown value needs to be calculated.

Using the initial values listed below velocity change was solved for using previous derivations to solve for the total change in velocity needed for a chaser and a target satellite to rendezvous:

Take a satellite in a 6600 km orbit and a chaser with a relative state where the relative radius is shown by{δr0} = $(1, 1, 1)^T$ (km) and the relative velocity is shown by {δv− 0} = $(0, 0, 5)^T$ (m/s) [1]

To solve the following problem, the Clohessy-Wiltshire equations and matrices were used to derive a solution.

Key:

Symbol	Meaning
μ	Gravitation Constant
r	Radius
T	Time Period
ΔV1	First Change in Velocity
ΔV2	Second Change in Velocity

Example Solution to Listed Problem:

Considering the rendezvous problem between a spacecraft and a satellite in orbit with the foci as the Earth. The objective is to derive the total change in velocity for the chaser so that it is able to rendezvous with the satellite which is in a fixed orbit with the condition that the rendezvous is completed within ⅓ of the spacecraft's orbital period.

The following equation will first give us the velocity of the spacecraft which has been defined as V in this equation. This is done by taking the square root of the gravitational constant (represented by symbol μ) divided by the radius of the spacecraft represented by symbol r.

$$V = \sqrt{\frac{\mu}{r}} \tag{55}$$

The symbol n is given as hertz or seconds inverse $\left(s^{-1}\right)$ which is calculated by using the velocity (V) divided by the radius of the spacecraft. This value will be used in determining the total time taken needed to perform the movement which is a requirement of the given problem.

$$n = \frac{V}{r} \tag{56}$$

The given symbol "T" represents the total time taken for the spacecraft to make one complete orbit around the Earth which is derived from using the formula above which uses the previous constant "n." Keep in mind that the value that is being used for time here is seconds.

$$T = \frac{2\pi}{n}$$
(57)

Using the value for the orbital period the time needed for the spacecraft and chaser to rendezvous can be determined, which is a set requirement of the problem and from this point it is possible to determine the values of our Clohessy-Wiltshire matrices.

$$t = \frac{T}{3}$$
(58)

Below are the formulas for the Clohessy-Wiltshire matrices which once the values of "n" and "t" are substituted in can be used to derive other values that can then be used in order to derive individual changes in velocity.

$$\Phi_{rr}(t) = \begin{bmatrix} 4- 3\cos(nt) & 0 & 0 \\ 6(\sin(nt) - nt) & 1 & 0 \\ 0 & 0 & \cos(nt) \end{bmatrix}$$
(59)

$$\Phi_{rv}(t) = \begin{bmatrix} \frac{1}{n}\sin(nt) & 0 & 0 \\ 6(\sin(nt) - nt) & 1 & 0 \\ 0 & 0 & \cos(nt) \end{bmatrix}$$
(60)

$$\Phi_{vr}(t) = \begin{bmatrix} 3n\sin(nt) & 0 & 0 \\ 6n(\cos(nt) - 1) & 0 & 0 \\ 0 & 0 & -n\sin(nt) \end{bmatrix}$$
(61)

$$\Phi_{vv}(t) =$$

$$\begin{bmatrix} cos\,(nt) & 2\sin(nt) & 0 \\ -2\sin(nt) & 4\cos(nt)-3 & 0 \\ 0 & 0 & \cos(nt) \end{bmatrix}$$

(62)

Using the values given in the Clohessy matrices and using the set values for δ_{rf} and δ_{r0} the equation can be rearranged in order to solve for $\delta_{v0}+$ which will then enable us to get ΔV_1

$$\delta_{rf} = \left(\Phi_{rr}\right)\left(\delta_{r0}\right) + \left(\Phi_{rv}\right)\left(\delta_{v0}+\right)$$

(63)

Using this equation, one would then be able to derive the first change in velocity. Additionally using ΔV_1 it will then be possible to calculate the initial velocity transfer.

$$\Delta V_1 = (\delta_{v0}+) - (\delta_{v0}-)$$

(64)

First the values for the Clohessy Wiltshire matrices are substituted into the given equation. Now using this equation ΔV_2 can be solved for by employing a similar method that has been used in order to derive ΔV_1

$$\delta_{vf} - \quad = (\Phi_{vr})(\delta_{r0}) + (\Phi_{vv})(\delta_{v0}+)$$

(65)

Now that $(\delta_{vf}+)$ has been declared as shown below this value can be substituted into the next step and begin to start solving for ΔV_2 .

.

$$(\delta_{vf}+) \;=0$$

(66)

After solving for ΔV_2 by substituting the values into the equation we can begin to calculate the total change in velocity which is the final requirement for the problem. After calculating ΔV_2 in matrix form it is required that we find the absolute value as adding ΔV_1 and ΔV_2 together will result in a matrix. After converting to a single value the units need to be changed from km/s to m/s.

$$\Delta V_2 = (\delta_{vf} +) - (\delta_{vf} -) \tag{67}$$

$$\Delta V_{total} = \Delta V_1 + \Delta V_2 \tag{68}$$

$$\Delta V_{total} = 6.21\ m/s \tag{69}$$

To be able to visualize the problem from a 3-D point of view the images below represent the trajectory of the chaser and its changes.

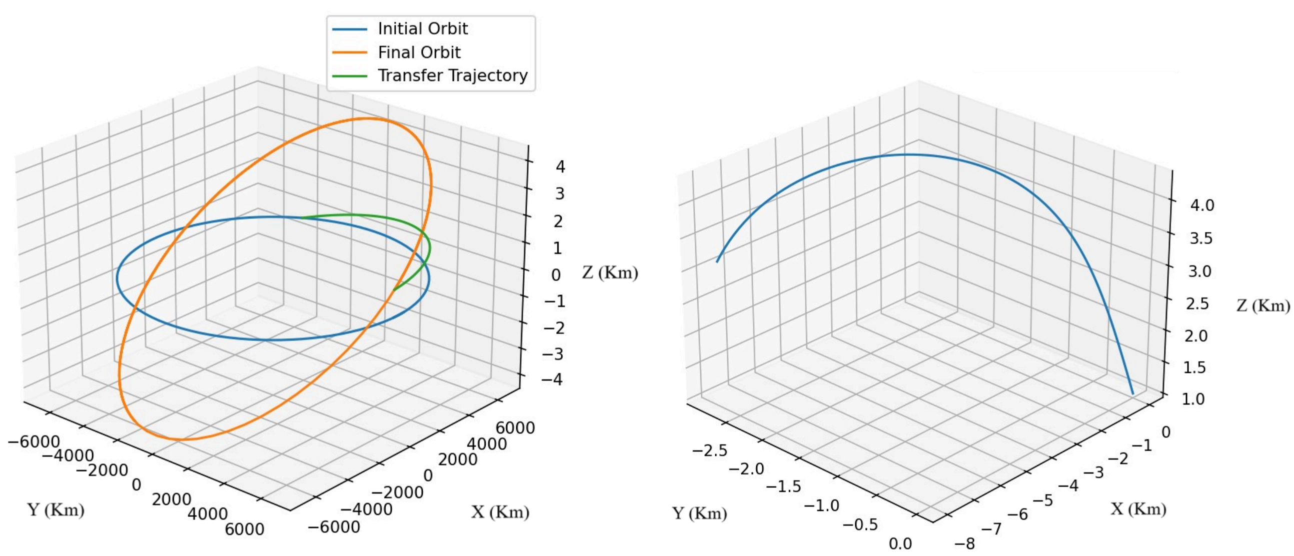

Figure 3: Rendezvous Trajectories: (a) Inertial View (b) Initial orbit relative trajectory

Conclusion

In conclusion, this research paper delved into the derivation of a satellite rendezvous scheme, exploring fundamental concepts such as the Hohmann transfer and the Clohessy-Wiltshire equations. By understanding these principles, valuable insights into the dynamics and mechanics of satellite rendezvous maneuvers were gained, crucial for space missions and satellite operations. The derivation of the Hohmann transfer provides a foundational understanding of the optimal orbital maneuver required to transfer a satellite from one circular orbit to another. Through mathematical analysis, key parameters influencing transfer orbits were uncovered, such as delta-v requirements. This knowledge forms the cornerstone for planning interplanetary missions and optimizing satellite trajectories. Furthermore, the exploration of the Clohessy-Wiltshire equations offered a deeper insight into the dynamics of relative motion between two satellites during rendezvous maneuvers. By employing a perturbation approach, we derived a set of differential equations describing the relative position and velocity of the chaser satellite with respect to the target satellite. These equations serve as a powerful tool for simulating and predicting satellite rendezvous scenarios, enabling precise control and coordination of space missions. Finally, through a practical application of these derivations, a specific problem was demonstrated related to satellite rendezvous. By applying the derived equations and principles, a systematic approach to planning and executing a rendezvous maneuver was formulated, taking into account factors such as orbital dynamics, and mission objectives. In essence, the derivation of a satellite rendezvous scheme presented in this research paper enhances our theoretical understanding of orbital mechanics and offers practical insights applicable to real-world space missions. Continuing to explore and expand our presence in space, the knowledge gained from this study will be invaluable for advancing satellite technology, enabling more efficient and precise rendezvous operations in the future.

Appendix

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Author

Raaid Ahmed Baloch is a Grade 11 student studying in Brighton College Dubai and is interested in Aerospace Engineering and is looking to pursue a career in the field.